

EXHIBIT 9

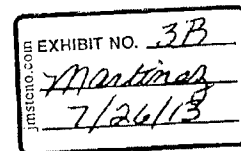
Phoenix College
Fall 2009

MAT 182

Precalculus

Trigonometry Section
Lecture Notes

Dr. Cleopatria Martínez



Chapter 1 – Trigonometry1.1 Angles and their Measure , Basic Angles, Degrees^o Minutes' Seconds", Degrees, Radians, arc length, area of a sector, linear speed, angular speed

Vocabulary: Angle, ray, line, segment, standard position of an angle, initial side,

A segment is a set of points with a starting point and an ending point.

A line is a set of points that go forever in two directions.

A ray is a set of points that start at one point then go forever.

An angle is two rays with the same starting point.

The vertex is the point at the corner of the angle.

The initial side is the ray where the angle starts.

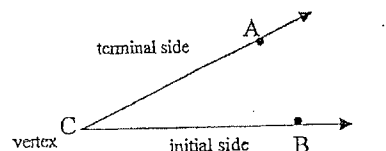
The terminal side is the ray where the angle ends.

A positive angle opens counter-clockwise.

A negative angle opens clockwise.

The name of the angle can be the vertex, like angle C i.e. $\angle C$ or by using three letters with the vertex in the middle i.e. $\angle ACB$ or $\angle BCA$.

Standard position has the vertex at the origin and the initial side on the positive x-axis.



The size of an angle is measured by degrees. A full rotation is 360° i.e. 360 degrees.

Degrees^o Minutes' Seconds" are used to describe the size of an angle.

One degree, written 1° , represents $\frac{1}{360}$ of a full rotation.

One minute, written $1'$, represents $\frac{1}{60}$ of a degree.

A degree can be divided into 60 parts called 60 minutes i.e. $60'$.

One second written $1''$, represents $\frac{1}{60}$ of a minute.

A degree can be divided into 60 parts called 60 seconds i.e. $60''$.

An acute angle has between 0 degrees and 90 degrees.

A right angle has exactly 90 degrees.

An obtuse angle has between 90 degrees and 180 degrees.

A straight angle has exactly 180 degrees.

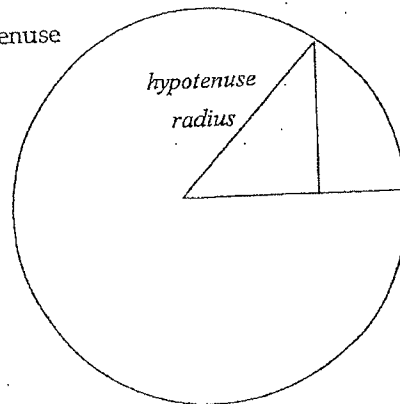
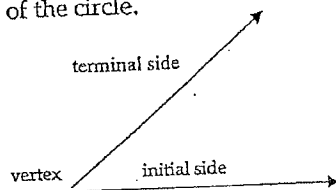
A reflexive angle has between 180 degrees and 360 degrees.

A complete rotation gives an angle measuring 360 degrees.

Complementary angles are two angles which add up to 90 degrees.

Supplementary angles are two angles which add up to 180 degrees.

Note the right triangle inscribed in the circle.
The terminal side of the angle is not only the hypotenuse of the right triangle but it is also the radius of the circle.

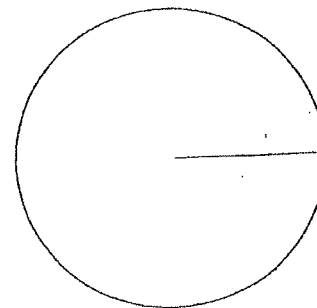


$$1 \text{ degree}^{\circ} = 60 \text{ minutes} = 59 \text{ minutes} + 60 \text{ seconds}$$

$$1 \text{ minute}' = 1' = \left(\frac{1}{60}\right)^{\circ}$$

$$1 \text{ second}'' = 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{60} \square \frac{1}{60}\right)^{\circ}$$

$$1 \text{ minute}' = 60 \text{ seconds}$$



Change Degrees-Minutes-Seconds into Degrees in decimal form.

Give the decimal answer accurate to two decimal places.

Example: Change $50^{\circ} 6' 21''$ into degrees. Answer:

$$50^{\circ} + 6' + 21''$$

$$50^{\circ} + 6 \left(\frac{1}{60}\right)^{\circ} + 21 \left(\frac{1}{60}\right)'$$

$$50^{\circ} + \frac{6}{1} \left(\frac{1}{60}\right)^{\circ} + \frac{21}{1} \left(\frac{1}{60} \square \frac{1}{60}\right)^{\circ}$$

$$50^{\circ} + \left(\frac{1}{10}\right)^{\circ} + \frac{3.7}{1} \left(\frac{1}{3.20 \cdot 60}\right)^{\circ}$$

$$50^{\circ} + .1^{\circ} + \left(\frac{7}{1200}\right)^{\circ}$$

$$\approx 50^{\circ} + .1^{\circ} + .005833^{\circ}$$

$$50.105833^{\circ} \text{ or } 50.11^{\circ}$$

Change Degrees into Degrees° Minutes' Seconds"Example: Change 21.256° into Degrees° Minutes' Seconds"

Answer:

$$21^\circ + .256^\circ$$

$$21^\circ + .256(60)'$$

$$21^\circ + 15.36'$$

$$21^\circ + 15' + .36'$$

$$21^\circ + 15' + .36(60)''$$

$$21^\circ + 15' + 21.6''$$

$$\approx 21^\circ 15' 22''$$

Example: Change 61.24° into DMS

Answer:

In many applications, like describing the exact location of a star or the exact position of a boat at sea, angles measured in degrees, minutes, and seconds are used. These are usually changed to degrees in decimal form.

1.1 Angles and their Measure Homework

Lial page 7 problems 1-21 odd, 23-28

Sullivan:

In problems 11-21, draw each angle.

$$11. 30^\circ \quad 13. 135^\circ \quad 15. 450^\circ \quad 17. \frac{3\pi}{4} \quad 19. -\frac{\pi}{6} \quad 21. \frac{16\pi}{3}$$

Convert each angle to a decimal in degrees. Round your answer to two decimal places.

$$23. 40^\circ 10' 25'' \quad 25. 1^\circ 2' 3'' \quad 27. 9^\circ 9' 9''$$

Convert each angle to $D^\circ M' S''$ form. Round your answer to the nearest second.

$$29. 40.32^\circ \quad 31. 18.255^\circ \quad 33. 19.99^\circ$$

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Plane Trigonometry Outline

Due Date	<u>Chapter 3 – Trigonometric Identities</u>	Page
_____	3.1.14 Trigonometric Identities (30) _____	53
_____	3.2.15 Sum & Differences Formulas (35) _____	57
_____	3.3.16 Double-angle & Power Reducing Formulas (35) _____	63
_____	3.4. Test on Trigonometric Identities --(no homework)_____	67

Chapter 4 – Advanced Trigonometry

_____	4.1.17 Trigonometric Equations (56) _____	69
_____	4.2.18 Applications involving Right Triangles (18) _____	73
_____	4.3.19 The Law of Sines (17) _____	75
_____	4.4.20 The Law of Cosines (16) _____	79

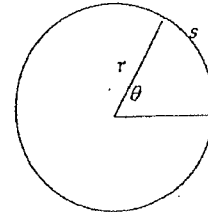
MAT 182 Final Exam Wednesday, May 12
 10:00 - 11:50 am in Room B210

1.2 Angles in Degrees and Angles in Radians

In calculus, angles are measured using radians.

A central angle is an angle whose vertex is at the center of a circle.

One radian is the measure of a central angle θ that cuts an arc s which is as long as the radius r of the circle: $s = r$
That angle is one radian, around 57° .



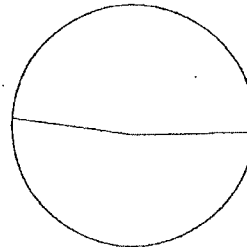
*If the angle is 1 radian, then the length of the arc is also 1 radian:
ie when the arc length = radius, then the angle is 1 radian or about 57° .*

If an angle θ is in degrees, use the number and the degree symbol when writing a trigonometric function like $\sin 30^\circ$ and $\tan 45^\circ$. If an angle θ is measured in radians, then no symbol is used to represent radians when writing a trigonometric function like $\cos \pi$ and $\sec \frac{\pi}{3}$.

Greek letters: α alpha β beta θ theta ϕ phi ω omega $\pi = \text{pi} \approx 3.14159265$ radians

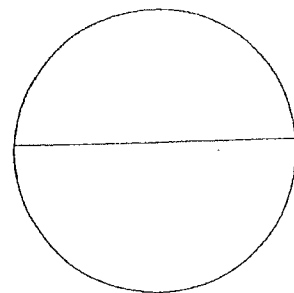
1 revolution = 2π radians

$\frac{1}{2}$ revolution = π radians

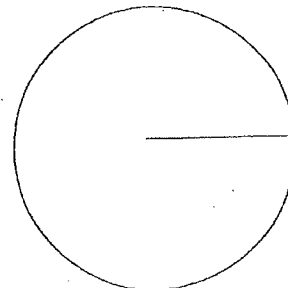


Divide π by 4 and label the angle.

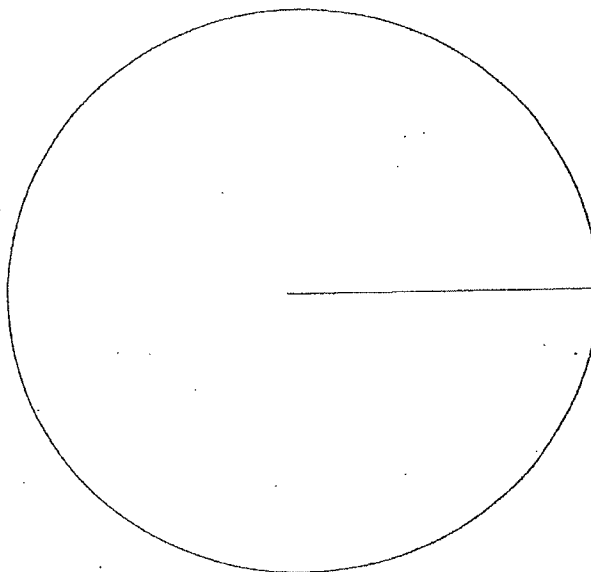
$\frac{1\pi}{4}$ $\frac{2\pi}{4}$ $\frac{3\pi}{4}$ $\frac{4\pi}{4}$



Divide π by 3 then divide π by 6.



Use the circle to mark the degree and radian measure of common angles.
 Divide the circle into multiples of 30° , then divide the circle into multiples of 45° .
 Transfer this information to the table provided.



Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
---------	-----------	------------	------------	------------	------------	-------------	-------------	-------------	-------------

Radians

Degrees	210°	225°	240°	270°	300°	315°	330°	360°
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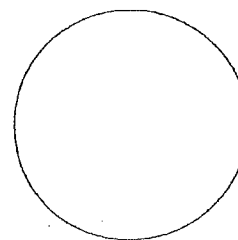
Radians

Change Degrees into radians:

$$180 \text{ degrees} = \pi \text{ radians}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$$

Solve for 1 degree by
dividing both sides by 180



Change Radians into degrees

$$\pi \text{ radians} = 180 \text{ degrees}$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \approx 57.3^\circ$$

Solve for 1 radian by
dividing both sides by π

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Example: Convert 60° into radians.

Answer: $60^\circ = 60(1 \text{ degree}) = \frac{60}{1} \square \frac{\pi}{180} = \frac{\pi}{3} \text{ radians}$

Example: Change 107° into radians.

Example: Convert $\frac{\pi}{6}$ radians into degrees.

Answer: $\frac{\pi}{6} \text{ radians} = \frac{\pi}{6} (1 \text{ radian}) = \frac{\pi}{6} \square \frac{180}{\pi} \text{ degrees} = 30 \text{ degrees ie } 30^\circ$

Example: Change 3 radians into degrees.

1.2 Angles in Degrees and Angles in Radians Homework

Convert each angle in degrees to radians. Express your answer as a multiple of π .35. 30° 37. 240° 39. -60° 41. 180° 43. -135° 45. -90°

Convert each angle in radians to degrees.

47. $\frac{\pi}{3}$ 49. $-\frac{5\pi}{4}$ 51. $\frac{\pi}{2}$ 53. $\frac{\pi}{12}$ 55. $-\frac{\pi}{2}$ 57. $-\frac{\pi}{6}$

Convert each angle in degrees to radians. Round answer to two decimal places.

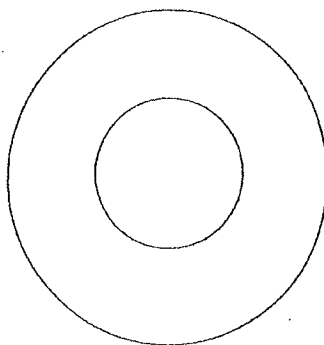
59. 17° 61. -40° 63. 125°

Convert each angle in radians to degrees. Round answer to two decimal places.

65. 3.14 67. 2 69. 6.32

1.3 Arc length, Area, and Velocity

Arc Length: The length of an arc l made by a central angle is found by multiplying the radius times the angle measure. $l = r\theta$



$$\frac{\angle \text{ of Big circle } \theta}{\angle \text{ of small circle "1" }} = \frac{\text{Big arc length } l}{\text{small arc length "r"}}$$

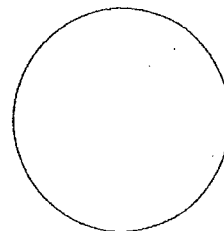
If the small circle's angle is 1 radian (ie 57°), then the length of the arc of the small circle is r , it's own radius.

$$\text{In other words, } \frac{\theta}{1} = \frac{\text{length of arc}}{r}$$

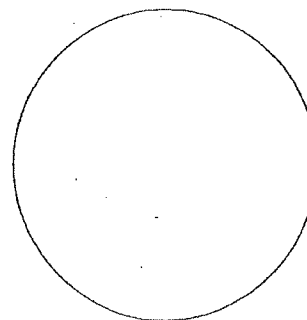
Solving this equation for the length of the arc l gives us the equation for arc length, $l = r\theta$

Example: Find the **length of the arc** of a circle whose radius is 2 meters and the central angle is .25 radian (ie $\frac{1}{4}$ radian).

Answer: The arc length $l = r\theta$
 $s = (2\text{meters})(.25)$
 $s = (2)(\frac{1}{4})$
 $s = .5 \text{ meter or } \frac{1}{2} \text{ meter.}$

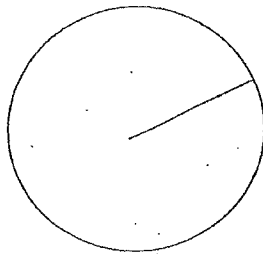


Example: Find the **length of the arc** of a circle whose radius is 10 meters and the central angle is .5 radian (ie $\frac{1}{2}$ radian).



Area of a sector $A = \frac{1}{2}r^2\theta$

A , the area of a slice of pie (ie a sector of a circle formed by a central angle) is found by multiplying $\frac{1}{2}$ times the radius squared times the angle in radians.



$$\frac{\text{Big angle}}{\text{small angle}} = \frac{\text{Big area}}{\text{small area}}$$

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

Suppose the small angle is 2π . The area of the circle is πr^2

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

Solving this equation for the area A gives us the equation for area, $A = \frac{1}{2}r^2\theta$

Example: Find the area of the sector with radius 2 feet and an angle of 30° .

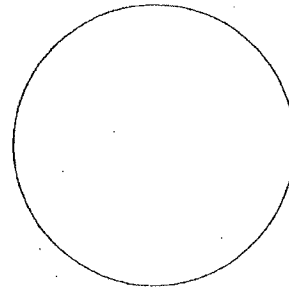
Answer: $A = \frac{1}{2}r^2\theta$ Draw the area of the sector.

$$A = \frac{1}{2} \left(\frac{2}{1} \right)^2 \left(\frac{\pi}{6} \right)$$

$$A = \frac{1}{2} \left(\frac{4}{1} \right) \left(\frac{\pi}{6} \right)$$

$$A = \frac{\pi}{3} \text{ square feet}$$

$$A \approx 1.05 \text{ square feet}$$



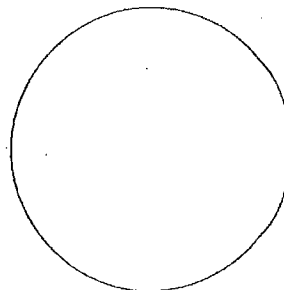
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Example: Find the area of the sector with radius 6 feet and an angle of 2 radians.

Answer: Write the formula and draw the area of the sector.



$$\text{Distance} = (\text{rate}) (\text{time}) \quad d = r \cup t \Rightarrow \text{rate} = \frac{\text{distance}}{\text{time}} \quad \text{ie } r = \frac{d}{t}$$

Linear speed = v of an object traveling in a circle is found by dividing the arc length s by the time t : $v = \frac{s}{t}$ length per unit of time, like feet per second or miles per hour

$$\text{linear speed} = v = \frac{\text{arc length}}{\text{unit of time}}$$

$$\text{Angular speed} = \omega = \frac{\text{angle in radians}}{\text{unit of time}}$$

Angular speed is how the turning rate of an engine is described. For instance, an engine idling at 900 rpm (revolutions per minute) is one that rotates at an angular speed of $900 \frac{\text{revolutions}}{\text{minute}} = \frac{900 \text{ revolutions}}{\text{minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} = 1800\pi \frac{\text{radians}}{\text{minute}}$

Since angular speed, ω , is given in terms of *revolutions* per unit of time, you must convert it to *radians* per unit of time.

There is an important relationship between linear speed and angular speed:

$$\text{linear speed} = v = \frac{s}{t} = r \cdot \omega \quad \text{so } v = r \cdot \omega$$

Remember that linear speed $v = \frac{s}{t}$ has the dimensions of length per unit of time (such as feet per second or miles per hour

In $v = r \cdot \omega$, the radius of the circular motion r has the same length dimension as s , ie if s is in feet then r is also in feet.

Summary of formulas

$$\text{distance} = (\text{rate})(\text{time}) \quad d = r \cup t \Rightarrow \text{rate} = \frac{\text{distance}}{\text{time}} \quad \text{ie } r = \frac{d}{t}$$

$$180 \text{ degrees} = \pi \text{ radians}$$

$$\text{Arc length } s = r \cdot \theta = (\text{the length of the radius})(\text{the size of the angle in radians})$$

$$\text{Area of a sector } A = \frac{1}{2} r^2 \theta = \left(\frac{1}{2}\right) (\text{the radius squared})(\text{the angle in radians})$$

$$\text{Linear speed } v = \frac{s}{t} = \frac{\text{length of the arc}}{\text{unit of time}} \quad \text{like feet per second or miles per hour}$$

$$\text{Also, } v = r \omega = (\text{radius of the circular motion})(\text{angular speed})$$

$$\text{Angular speed } \omega = \frac{\theta}{t} = \frac{\text{central angle in radians}}{\text{time elapsed}} = \text{radians per unit of time}$$

1.3 Arc length, Area, and Velocity Homework

L denotes the length of the arc of a circle of radius r subtended by the central angle θ . Find the missing quantity rounded to three decimal places.

$$71. r = 10 \text{ meters, } \theta = \frac{1}{2} \text{ radian, } L = ? \quad 73. \theta = \frac{1}{3} \text{ radian, } L = 2 \text{ feet, } r = ?$$

$$75. r = 5 \text{ miles, } L = 3 \text{ miles, } \theta = ? \quad 77. r = 2 \text{ inches, } \theta = 30^\circ, L = ?$$

In problems 79-85, A denotes the area of the sector of a circle of a radius r formed by the central angle θ . Find the missing quantity rounded to three decimal places.

$$79. r = 10 \text{ meters, } \theta = \frac{1}{2} \text{ radian, } A = ? \quad 81. \theta = \frac{1}{3} \text{ radian, } A = 2 \text{ square feet, } r = ?$$

$$83. r = 5 \text{ miles, } A = 3 \text{ square miles, } \theta = ? \quad 85. r = 2 \text{ inches, } \theta = 30^\circ, A = ?$$

87. Draw a circle with angle $\frac{\pi}{3}$ and radius 2 ft. Find the arc length L and the area A , both rounded to three decimal places.

88. Draw a circle with angle $\frac{\pi}{6}$ and radius 4 m. Find the arc length L and the area A , both rounded to three decimal places.

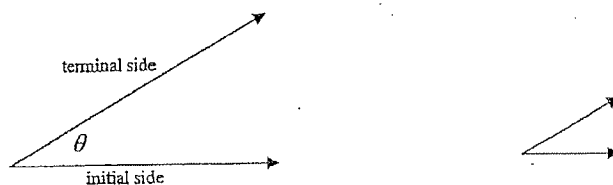
89. Draw a circle with angle 70° and radius 12 yds. Find the arc length L and the area A , both rounded to three decimal places.

90. Draw a circle with angle 50° and radius 9 cm. Find the arc length L and the area A , both rounded to three decimal places.

1.4 Right-Triangle Trigonometry

A right triangle has the famous Pythagorean Theorem related to it: $a^2 + b^2 = c^2$
 $(leg)^2 + (leg)^2 = (hypotenuse)^2$

Draw the rectangular axes so the *acute* angles shown below have the vertex of each angle at the origin. Form two right-triangles by dropping a perpendicular line from the tip of the terminal side to the x-axis. The triangles are similar triangles so they have proportionate sides. Label the sides of the larger triangle 9, 12, and 15, and the sides of the smaller triangle 3, 4, and 5.



The trigonometry functions are ratios of *the lengths* of two sides of a right triangle. Each ratio depends on the size of the angle θ and not the length of the sides of the triangle. Note that in each triangle, the ratio formed by the terminal side over the initial side is $\frac{5}{4}$.

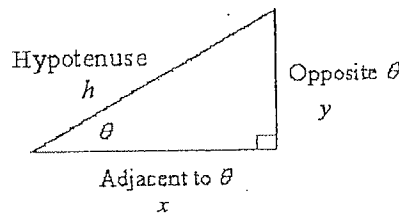
The six trigonometric functions are: $\sin \theta$ $\cos \theta$ $\tan \theta$ $\csc \theta$ $\sec \theta$ $\cot \theta$

$\sin \theta$ is read "the sine of theta."

You cannot separate them. The $\sin \theta$ is a ratio. However, \sin without θ is a "pecado," a sin, not a ratio. Therefore, always *write the trig function with the angle*.

$\sin \theta$


The angle is called the argument of the function.



The ratio called $\sin \theta$ is formed by the *length* of the side opposite the angle over the *length* of the side called the hypotenuse: $\sin \theta = \frac{\text{Opposite}}{\text{hypotenuse}}$

The ratio called $\cos \theta$ is formed by the *length* of the side adjacent (next to) the angle over the *length* of the side called the hypotenuse: $\cos \theta = \frac{\text{Adjacent}}{\text{hypotenuse}}$

The ratio called $\tan \theta$ is formed by the *length* of the side opposite the angle over the *length* of the side adjacent (next to) the angle: $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

When graphing, the $\sin \theta$ is referred to as the "y-coordinate" and the $\cos \theta$ is referred to as the "x-coordinate." Therefore, a point has the coordinates $(\cos \theta, \sin \theta)$ instead of (x, y) .

"O" and "y" are the side opposite the angle. "A" and "x" are the side adjacent (next) to the angle.

The hypotenuse "h" is also known as the radius "r" of the right triangle. Each ratio has a name that depends on the angle θ . The 6 ratios are formed as described below.

sine of $\theta = \frac{\text{Opposite}}{\text{hypotenuse}}$	cosine of $\theta = \frac{\text{Adjacent}}{\text{hypotenuse}}$	tangent of $\theta = \frac{\text{Opposite}}{\text{Adjacent}}$
$\sin \theta = \frac{O}{h} = \frac{y}{r}$	$\cos \theta = \frac{A}{h} = \frac{x}{r}$	$\tan \theta = \frac{O}{A} = \frac{y}{x}$
cosecant of $\theta = \frac{\text{hypotenuse}}{\text{Opposite}}$	secant of $\theta = \frac{\text{hypotenuse}}{\text{Adjacent}}$	cotangent of $\theta = \frac{\text{Adjacent}}{\text{Opposite}}$
$\csc \theta = \frac{h}{O} = \frac{r}{y}$	$\sec \theta = \frac{h}{A} = \frac{r}{x}$	$\cot \theta = \frac{A}{O} = \frac{x}{y}$

Notice that the reciprocal of the sine function is the cosecant function. The reciprocal identities are provided below.

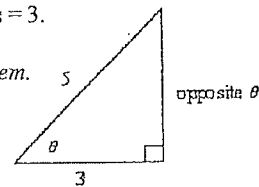
Reciprocal Identities $\csc \theta = \frac{1}{\sin \theta}$ $\sin \theta = \frac{1}{\csc \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\cos \theta = \frac{1}{\sec \theta}$

$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$ $\tan \theta = \frac{1}{\cot \theta} = \frac{\sin \theta}{\cos \theta}$ Memorize these reciprocal identities.

Example 1 Find the value of the six trigonometric functions of the angle θ in the triangle. We see that the hypotenuse = 5 and the adjacent side is 3.

To find the length of the opposite side, we use the Pythagorean Theorem.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (\text{leg})^2 + (\text{leg})^2 &= (\text{hypotenuse})^2 \\ (\text{adjacent})^2 + (\text{opposite})^2 &= (\text{hypotenuse})^2 \\ 3^2 + (\text{opposite})^2 &= 5^2 \\ (\text{opposite})^2 &= 25 - 9 = 16 \\ \text{opposite} &= 4 \end{aligned}$$



Now that we know the lengths of the three sides, we can find the six trigonometric functions:

$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5} & \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5} & \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3} \\ \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{4} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{3} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} = \frac{3}{4} \end{aligned}$$

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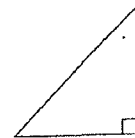
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EXAMPLE 2 $\sin \theta = \frac{\sqrt{5}}{5}$ Find the remaining 5 trigonometric functions of θ .

Solution: $\cos \theta =$

Label the sides of a right triangle:



$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \frac{1}{2}$$

$$a^2 + b^2 = c^2$$

$$\csc \theta = \frac{1}{\sin \theta} =$$

$$\sec \theta = \frac{1}{\cos \theta} =$$

$$\cot \theta = \frac{1}{\tan \theta} =$$

Steps for Finding the Values of 5 Trigonometric Functions When One is Known

- Step 1: Draw a right triangle showing the angle θ and label two sides of the triangle.
 Step 2: Find the length of the third side by using the Pythagorean Theorem.
 Step 3: Use the definitions to find the value of the remaining 5 trigonometric functions.

Example 4 $\sin \theta = \frac{1}{3}$. Find the exact value of the remaining five trigonometric functions.

- Solution:** 1) Draw and label the sides of a right triangle in Quadrant 1.
 2) Use the Pythagorean Theorem to find the adjacent side.

$$a^2 + b^2 = c^2$$

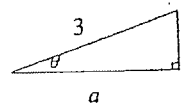
$$(\text{adjacent side})^2 + (\text{opposite side})^2 = (\text{hypotenuse})^2$$

$$(\text{adjacent})^2 + 1^2 = 3^2$$

$$(\text{adjacent})^2 + 1 = 9$$

$$(\text{adjacent})^2 = 8$$

$$\text{adjacent} = 2\sqrt{2}$$



- 3) Use the definitions to find the value of the remaining five trigonometric functions.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} =$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} =$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} =$$

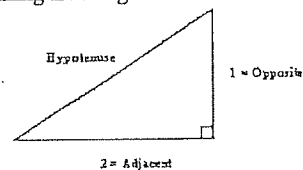
$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} =$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} =$$

$$\frac{2\sqrt{2}}{3} \quad \frac{\sqrt{2}}{4} \quad 3 \quad \frac{3\sqrt{2}}{4} \quad 2\sqrt{2}$$

Example 5 If $\tan \theta = \frac{1}{2}$, find the exact value of the remaining five trig functions.

1) Draw the right triangle where $\tan \theta = \frac{1}{2} = \frac{\text{opposite}}{\text{adjacent}}$



2) Use the Pythagorean Theorem to find the hypotenuse.

3) Now use the definitions to find the value of the other 5 trigonometric functions.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} =$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} =$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} =$$

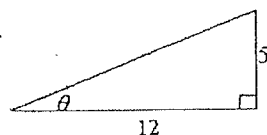
$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} =$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}} =$$

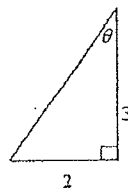
1.4 Right-Triangle Trigonometry Homework

Find the value of the six trigonometric functions in triangles 11-19.

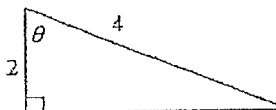
11.



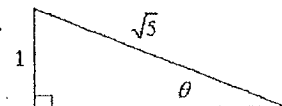
13.



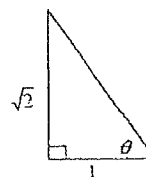
15.



17.



19.



For #21 and 23, use identities to find the value of the remaining trigonometric functions.

21. $\sin \theta = \frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$

23. $\sin \theta = \frac{2}{3}$, $\cos \theta = \frac{\sqrt{5}}{3}$

In problems 25-35, find the value of the remaining five trigonometric functions.

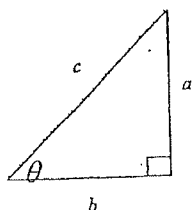
25. $\sin \theta = \frac{\sqrt{2}}{2}$ 27. $\cos \theta = \frac{1}{3}$ 29. $\tan \theta = \frac{1}{2}$ 31. $\sec \theta = 3$ 33. $\tan \theta = \sqrt{2}$ 35. $\csc \theta = 2$

1.5 The Pythagorean Identity and Complementary Functions

Pythagorean Identities

You now know enough so we can derive the Pythagorean Identities.

The Pythagorean Theorem states that $a^2 + b^2 = c^2$, which we can write as
 $(\text{opposite})^2 + (\text{adjacent})^2 = (\text{hypotenuse})^2$



Dividing each side by the $(\text{hypotenuse})^2$ we get

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} \quad \text{or} \quad \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$$(\sin \theta)^2 + (\cos \theta)^2 = 1$$

The Pythagorean Identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

Divide the Pythagorean Identity by $\cos^2 \theta$ and simplify.

$$\sin^2 \theta + \cos^2 \theta = 1$$

Divide the Pythagorean Identity by $\sin^2 \theta$ and simplify.

$$\sin^2 \theta + \cos^2 \theta = 1$$

PYTHAGOREAN IDENTITIES

Commit these Pythagorean Identities to memory.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

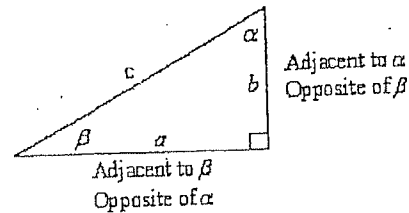
Recall that two acute angles are called **complementary** if they add up to 90 degrees. Also, the three angles of any triangle add up to 180° . So in a right triangle, the two acute angles are complementary angles. Additionally:

The sine and cosine are called **cofunctions**.

The cosecant and secant are cofunctions.

The tangent and cotangent are cofunctions.

Which leads us to ↓



The Complementary Angle Theorem: Cofunctions of complementary angles are *equal*.

$$\sin \beta = \frac{\text{opposite}}{\text{hypotenuse}} = \cos \alpha$$

$$\cos \beta = \frac{\text{adjacent}}{\text{hypotenuse}} = \sin \alpha$$

$$\tan \beta = \frac{\text{opposite}}{\text{adjacent}} = \cot \alpha$$

$$\csc \beta = \frac{\text{hypotenuse}}{\text{opposite}} = \sec \alpha$$

$$\sec \beta = \frac{\text{hypotenuse}}{\text{adjacent}} = \csc \alpha$$

$$\cot \beta = \frac{\text{adjacent}}{\text{opposite}} = \tan \alpha$$

Complementary angles

$$\sin 30^\circ = \cos 60^\circ$$

Cofunctions

Complementary angles

$$\tan 40^\circ = \cot 50^\circ$$

Cofunctions

Complementary angles

$$\sec 80^\circ = \csc 10^\circ$$

Cofunctions

Because of these relationships, the sine and cosine, the tangent and cotangent, and the secant and cosecant are called **cofunctions**.

Example 6 Using the Complementary Angle Theorem, find the corresponding cofunctions.

$$\sin 62^\circ = \cos (90^\circ - 62^\circ) = \cos \underline{\hspace{2cm}}$$

$$\tan \frac{\pi}{2} = \cot \underline{\hspace{2cm}}$$

$$\cos \frac{\pi}{4} = \sin \underline{\hspace{2cm}}$$

$$\csc \frac{\pi}{6} = \sec \underline{\hspace{2cm}}$$

Example 7 Find the value of each expression. Do not use a calculator.

(a) $\sec 28^\circ - \csc 62^\circ$ (b) $\frac{\sin 35^\circ}{\cos 55^\circ}$

1.5 The Pythagorean Identity and Complementary Functions Homework

In Problems 37-53, find the exact value of each expression. *Do not use a calculator.*

37. $\sin^2 20^\circ + \cos^2 20^\circ$ 39. $\sin 80^\circ - \cos 52^\circ$ 41. $\tan 50^\circ - \frac{\sin 50^\circ}{\cos 50^\circ}$ 43. $\sin 38^\circ - \cos 52^\circ$

45. $\frac{\cos 10^\circ}{\sin 80^\circ}$ 47. $1 - \cos^2 20^\circ - \cos^2 70^\circ$ 49. $\tan 20^\circ - \frac{\cos 70^\circ}{\cos 20^\circ}$

51. $\tan 35^\circ \cdot \sec 55^\circ \cdot \cos 35^\circ$ 53. $\cos 35^\circ \sin 55^\circ + \cos 55^\circ \sin 35^\circ$

55. Given $\sin 30^\circ = \frac{1}{2}$, use trigonometric identities to find the exact value of

(a) $\cos 60^\circ$ (b) $\cos^2 30^\circ$ (c) $\csc \frac{\pi}{6}$ (d) $\sec \frac{\pi}{3}$

57. Given $\tan \theta = 4$, use trigonometric identities to find the exact value of

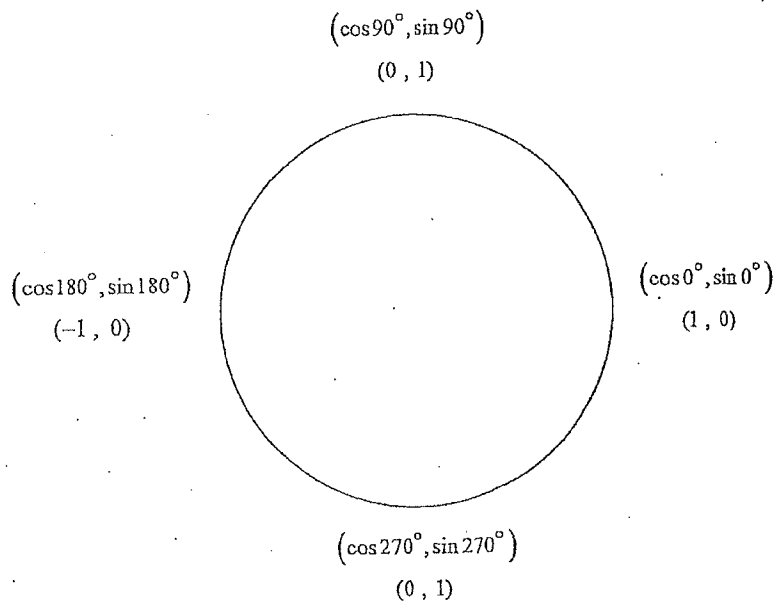
(a) $\sec^2 \theta$ (b) $\cot \theta$ (c) $\cot \left(\frac{\pi}{2} - \theta \right)$ (d) $\csc^2 \theta$

59. Given $\csc \theta = 4$, use trigonometric identities to find the exact value of

(a) $\sin \theta$ (b) $\cot^2 \theta$ (c) $\sec(90^\circ - \theta)$ (d) $\sec^2 \theta$

1.6 Computing the Values of Trig Functions for Special Triangles

The special angles in trigonometry are $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$
 In radian measure these angles are $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{4}, 2\pi$

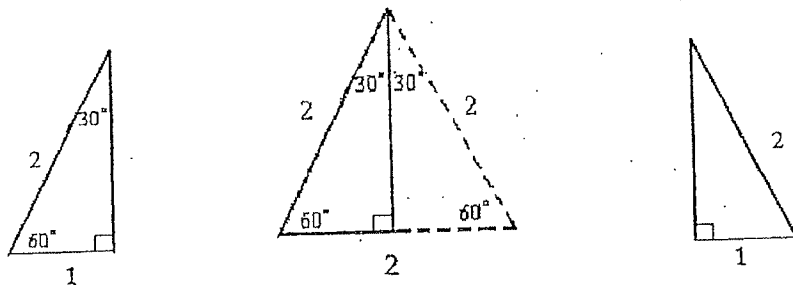


Commit to memory the values of the 6 trigonometric functions
 for angles $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

$\sin 0^\circ = 0$	$\sin 90^\circ = 1$	$\sin 180^\circ = 0$	$\sin 270^\circ = -1$	$\sin 360^\circ = 0$
$\cos 0^\circ = 1$	$\cos 90^\circ = 0$	$\cos 180^\circ = -1$	$\cos 270^\circ = 0$	$\cos 360^\circ = 1$
$\csc 0^\circ = \text{undef}$	$\csc 90^\circ = 1$	$\csc 180^\circ = \text{undef}$	$\csc 270^\circ = -1$	$\csc 360^\circ = \text{undef}$
$\sec 0^\circ = 1$	$\sec 90^\circ = \text{undef}$	$\sec 180^\circ = -1$	$\sec 270^\circ = \text{undef}$	$\sec 360^\circ = 1$
$\tan 0^\circ = 0$	$\tan 90^\circ = \text{undef}$	$\tan 180^\circ = 0$	$\tan 270^\circ = \text{undef}$	$\tan 360^\circ = 0$
$\cot 0^\circ = \text{undef}$	$\cot 90^\circ = 0$	$\cot 180^\circ = \text{undef}$	$\cot 270^\circ = 0$	$\cot 360^\circ = \text{undef}$

Consider an equilateral triangle cut in half so the top 60° angle makes two 30° angles.

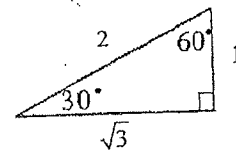
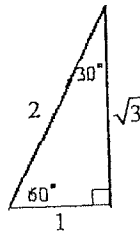
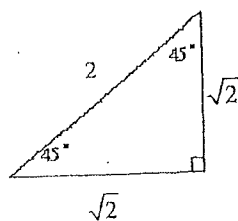
Each right triangle is now a special triangle known as a 30° - 60° - 90° triangle.



The sides of the right triangles measure 1, 2, $\sqrt{3}$. The hypotenuse measures 2.
The side opposite the 30° measures 1, and the side opposite the 60° angle measures $\sqrt{3}$.

Commit to memory the trigonometric function values of the two very special triangles often used in trigonometry. They are the

45° 45° 90° or $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}$ and the 30° 60° 90° or $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$



$$\sin 45^\circ = \frac{\sqrt{2}}{2} \quad \csc 45^\circ = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \quad \sec 45^\circ = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

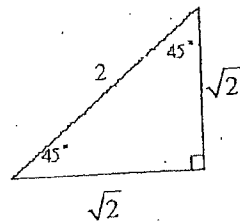
$$\tan 45^\circ = \frac{\sqrt{2}}{\sqrt{2}} = 1 \quad \cot 45^\circ = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan 60^\circ = \sqrt{3}$$

Example 1 Find the values of the six trigonometric functions of $\frac{\pi}{4} = 45^\circ$.

Solution Draw the triangle and label the sides and the angles.



$$\sin \frac{\pi}{4} = \sin 45^\circ =$$

$$\cos \frac{\pi}{4} =$$

$$\sec \frac{\pi}{4} =$$

$$\csc \frac{\pi}{4} =$$

$$\tan \frac{\pi}{4} =$$

$$\cot \frac{\pi}{4} =$$

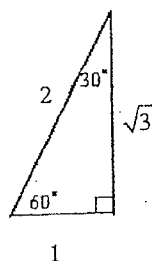
Example 2 Find the exact value of each expression.

a) $(\sin 45^\circ)(\tan 45^\circ)$

b) $(\sec \frac{\pi}{4})(\cot \frac{\pi}{4})$

Example 3 Find the Exact Values of the six trigonometric functions of $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$

Solution: Form a right triangle and label the sides and the angles appropriately.



$$\sin \frac{\pi}{6} =$$

$$\cos \frac{\pi}{3} =$$

$$\cos \frac{\pi}{6} =$$

$$\sin \frac{\pi}{3} =$$

$$\tan \frac{\pi}{6} =$$

$$\cot \frac{\pi}{3} =$$

$$\csc \frac{\pi}{6} =$$

$$\sec \frac{\pi}{3} =$$

$$\sec \frac{\pi}{6} =$$

$$\csc \frac{\pi}{3} =$$

$$\cot \frac{\pi}{6} =$$

$$\tan \frac{\pi}{3} =$$

Example 4 Find the exact value of each expression.

(a) $\sin 45^\circ \cos 30^\circ$ (b) $\tan \frac{\pi}{4} - \sin \frac{\pi}{3}$ (c) $\tan^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{4}$

Example 5 Use a calculator to find the approximate value (to two decimal places) of:

(a) $\cos 48^\circ$ (b) $\csc 21^\circ$ (c) $\tan \frac{\pi}{12}$

Solution Since these are not the special angles, we will use the calculator as follows.

(a) First, we set the MODE on the calculator to receive degrees.

(b) Most calculators do not have a csc key. However, the $\csc 21^\circ$ is $\frac{1}{\cos 21^\circ}$.

(c) Set the MODE on the calculator to receive radians.

1.6 Computing the Values of Trig Functions for Special Triangles Homework

In problems 7-16, $f(\theta) = \sin \theta$ and $g(\theta) = \cos \theta$. Find the value of each expression if $\theta = 60^\circ$. Do not use a calculator.

7. $f(\theta)$ 9. $f\left(\frac{\theta}{2}\right)$ 11. $[f(\theta)]^2$ 13. $2f(\theta)$ 15. $\frac{f(\theta)}{2}$

In problems 17-27, find the exact value of each expression. Do not use calculator.

17. $4 \cos 45^\circ - 2 \sin 45^\circ$ 19. $6 \tan 45^\circ - 8 \cos 60^\circ$ 21. $\sec \frac{\pi}{4} + 2 \csc \frac{\pi}{3}$

23. $\sec^2 \frac{\pi}{6} - 4$ 25. $\sin^2 30^\circ + \cos^2 60^\circ$ 27. $1 - \cos^2 30^\circ - \cos^2 60^\circ$

In problems 29-45, use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

29. $\sin 28^\circ$ 31. $\tan 21^\circ$ 33. $\sec 41^\circ$ 35. $\sin \frac{\pi}{10}$ 37. $\tan \frac{5\pi}{12}$

39. $\sec \frac{\pi}{12}$ 41. $\sin 1$ 43. $\sin 1^\circ$ 45. $\tan 0.3$

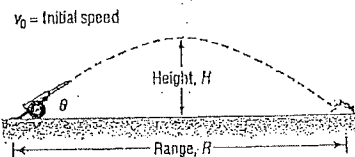
47. Applications and Extensions Find the range R and maximum height H rounded to 2 decimal places.

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found using the function

$$R(\theta) = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

Where $g \approx 32.2$ feet per second ≈ 9.8 meters per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 \sin^2 \theta}{2g}$$



Find the range R and maximum height H to two decimal places when the projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

1.7 Trigonometric Functions of General Angles

Example 1 Find the value of the six trigonometric functions of a positive angle θ if $(4, -3)$ is a point on its terminal side. Hint: Start by drawing the angle and the corresponding triangle. Label the sides.

Example 2 Find the values of the six trigonometric functions of

(a) $\theta = 0 = 0^\circ$ (b) $\theta = \frac{\pi}{2} = 90^\circ$ (c) $\theta = \pi = 180^\circ$ (d) $\theta = \frac{3\pi}{2} = 270^\circ$

Answers:

a) $\sin 0 = \sin 0^\circ = 0$

$\cos 0 = 1$

$\tan 0 = 0$

$\sec 0 = 1$

$\csc 0$ & $\cot 0$

are not defined

b) $\sin \frac{\pi}{2} = 1$

$\cos \frac{\pi}{2} = 0$

$\csc \frac{\pi}{2} = 1$

$\cot \frac{\pi}{2} = 0$

$\tan \frac{\pi}{2}$ and $\sec \frac{\pi}{2}$

are not defined

c) $\sin \pi = 0$

$\cos \pi = -1$

$\tan \pi = 0$

$\sec \pi = -1$

$\csc \pi$ & $\cot \pi$

are not defined

d) $\sin \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$

$\csc \frac{3\pi}{2} = -1$

$\cot \frac{3\pi}{2} = 0$

$\tan \frac{3\pi}{2}$ & $\sec \frac{3\pi}{2}$

are not defined

Example 3 a) $\sin 390^\circ$ b) $\cos 420^\circ$ c) $\tan \frac{9\pi}{4}$ d) $\sec\left(-\frac{7\pi}{4}\right)$ e) $\csc(-270^\circ)$

Hint: It is best to sketch the angle first.

a) The angle is coterminal with 30° .

$$\therefore \sin 390^\circ = \sin(360^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

b)

c)

d)

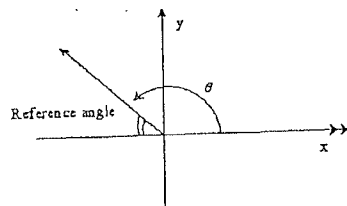
e)

Example 4 Name the Quadrant in Which the Angle Lies

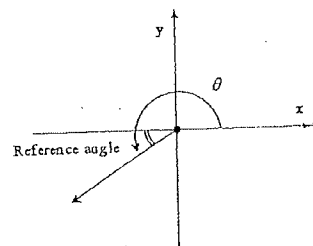
(a) $\sin \theta < 0$ and $\cos \theta < 0$

(b) $\sin \theta > 0$ and $\cos \theta < 0$

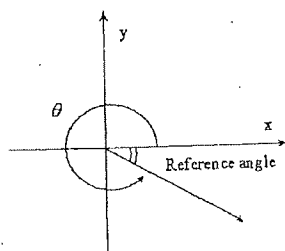
(c)



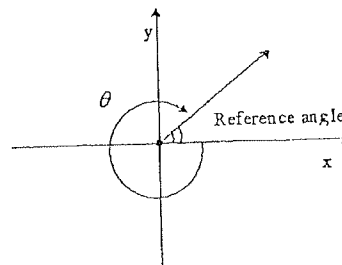
(d)



(e)



(f)

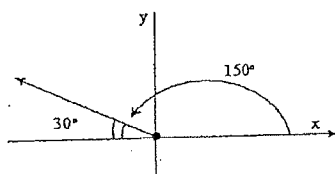


Example 5: Find the sine of the reference angle for each of the following angles:

- (a) 150° (b) -45° (c) $\frac{9\pi}{4}$ (d) $-\frac{5\pi}{6}$

Solution (a) $\sin 30^\circ = \frac{1}{2}$

(b)



(c)

(d)

Steps for finding the values of the trigonometric functions of an Angle

Step 1: If the angle θ is on the x-axis or y-axis, label a point on its terminal side, and name the trig values.
 Step 2: If the angle θ is in a quadrant, draw a triangle and label its sides, then name the trig values.

Example 6: Find the value of the following trigonometric functions.

- (a) $\sin 135^\circ$ (b) $\cos 600^\circ$ (c) $\cos \frac{17\pi}{6}$ (d) $\tan \left(\frac{\pi}{-3} \right)$

Example 7: Given that $\cos \theta = -\frac{2}{3}$, $\frac{\pi}{2} < \theta < \pi$, find the value of the remaining trig functions.

Example 8: If $\tan \theta = -4$ and $\sin \theta < 0$, find the value of the remaining trigonometric functions.

1.7 Trigonometric Functions of General Angles Homework

In problems 11-20, a point on the terminal side of an angle θ is given. Find the value of the six trigonometric functions.

11. $(-3, 4)$ 13. $(2, -3)$ 15. $(-3, -3)$ 17. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 19. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$

For #21-32, find the value of each expression. Do not use a calculator.

21. $\sin 405^\circ$ 23. $\tan 405^\circ$ 25. $\csc 450^\circ$ 27. $\cot 390^\circ$ 29. $\cos \frac{33\pi}{4}$ 31. $\tan(21\pi)$

In Problems 33-39, name the quadrant in which the angle θ lies.

33. $\sin \theta > 0, \cos \theta < 0$ 35. $\sin \theta < 0, \tan \theta < 0$ 37. $\cos \theta > 0, \cot \theta < 0$ 39. $\sec \theta < 0, \tan \theta > 0$

In Problems 41-57, name the reference angle of each angle.

41. -30° 43. 120°
45. 210° 47. $\frac{5\pi}{4}$ 49. $\frac{8\pi}{3}$ 51. 135° 53. $-\frac{2\pi}{3}$ 55. 440° 57. $\frac{15\pi}{4}$

For #59-88, find the value. Do not use the calculator.

59. $\sin 150^\circ$ 61. $\cos 315^\circ$ 63. $\sin 510^\circ$
65. $\cos(-45^\circ)$ 67. $\sec 240^\circ$ 69. $\cot 330^\circ$ 71. $\cos \frac{3\pi}{4}$ 73. $\cot \frac{7\pi}{6}$ 75. $\cos \frac{13\pi}{4}$
77. $\sin\left(-\frac{2\pi}{3}\right)$ 79. $\tan \frac{14\pi}{3}$ 81. $\csc(-315^\circ)$ 83. $\sin(8\pi)$ 85. $\tan(7\pi)$ 87. $\sec(-3\pi)$

In Problems 89-106, find the value of the remaining trigonometric functions.

89. $\sin \theta = \frac{12}{13}$, θ in Quadrant II 91. $\cos \theta = -\frac{4}{5}$, θ in Quadrant III
93. $\sin \theta = \frac{5}{13}$, $90^\circ < \theta < 180^\circ$ 95. $\cos \theta = -\frac{1}{3}$, $180^\circ < \theta < 270^\circ$
97. $\sin \theta = \frac{2}{3}$, $\tan \theta < 0$ 99. $\sec \theta = 2$, $\sin \theta < 0$ 101. $\tan \theta = \frac{3}{4}$, $\sin \theta < 0$
103. $\tan \theta = -\frac{1}{3}$, $\sin \theta > 0$ 105. $\csc \theta = -2$, $\tan \theta > 0$

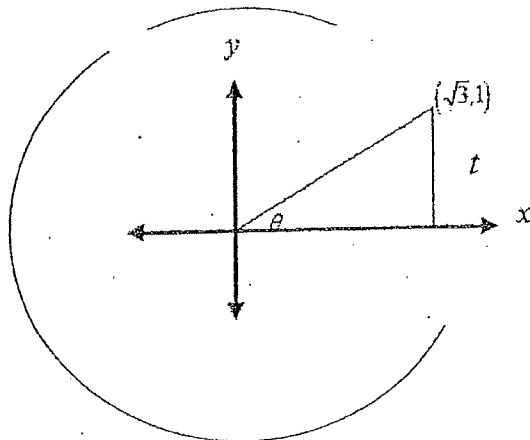
107. Find the exact value of $\sin 45^\circ + \sin 135^\circ + \sin 225^\circ + \sin 315^\circ$

109. If $f(\theta) = \sin \theta = 0.2$, find $f(\theta + \pi)$ 111. If $F(\theta) = \tan \theta = 3$, find $F(\theta + \pi)$.

113. If $\sin \theta = \frac{1}{5}$, find $\csc \theta$.

1.8 Circle Trigonometry

Let t be the length of the arc from the x -axis to the point $P = (\sqrt{3}, 1)$ on a circle.

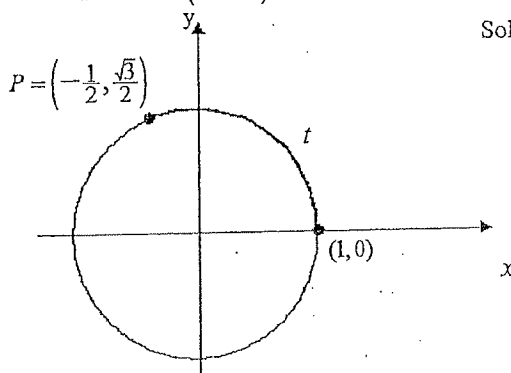


$$\theta = t \text{ radians}$$

$$\sin \theta = \sin t = \frac{1}{2}$$

$$\cos \theta = \cos t = \frac{\sqrt{3}}{2}$$

Example 1 $P = (-1, \sqrt{3})$ Find the value of the corresponding 6 trigonometric functions.



Solution: $\sin t = \frac{\sqrt{3}}{2}$

The unit circle has a radius of 1.

Domain & Range of Trigonometric Functions

Function	Domain is the angle	Range is the function
$f(\theta) = \sin \theta$	$\theta = \text{All real numbers}$	$\sin \theta = [-1 \text{ to } 1]$
$f(\theta) = \cos \theta$	$\theta = \text{All real numbers}$	$\cos \theta = [-1 \text{ to } 1]$
$f(\theta) = \tan \theta$	$\theta = (-\infty, +\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\tan \theta = (-\infty, +\infty)$
$f(\theta) = \csc \theta$	$\theta = (-\infty, +\infty)$, except integer multiples of π	$\csc \theta \leq -1, \csc \theta \geq 1$
$f(\theta) = \sec \theta$	$\theta = (-\infty, +\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\sec \theta \leq -1, \sec \theta \geq 1$
$f(\theta) = \cot \theta$	$\theta = (-\infty, +\infty)$, except integer multiples of π	$\cot \theta = (-\infty, +\infty)$

Periodic functions repeat after a certain time.

The trigonometric functions repeat so they are periodic functions.

2π is the repeating period for sine, cosine, cosecant, and secant.

$$\sin(\theta + 2\pi) = \sin \theta$$

$$\cos(\theta + 2\pi) = \cos \theta$$

$$\csc(\theta + 2\pi) = \csc \theta$$

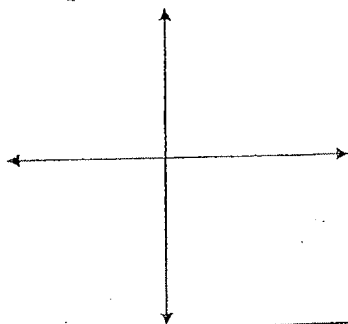
$$\sec(\theta + 2\pi) = \sec \theta$$

For tangent it is π .

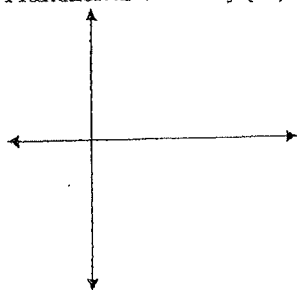
$$\tan(\theta + \pi) = \tan \theta$$

$$\cot(\theta + \pi) = \cot \theta$$

Example 2 Find the exact value of: a) $\sin 420^\circ$ b) $\tan \frac{5\pi}{4}$ c) $\cos \frac{11\pi}{4}$



Even functions are symmetric about the y-axis, odd functions about the origin.
Recall that a function is even if $f(-x) = f(x)$. The sign of the x changes but the y doesn't.
A function is odd if $-f(-x) = f(x)$. The sign of x and y changes, but the graph doesn't.



$$\cos\left(\frac{\pi}{6}\right)$$

$$\sin\left(\frac{\pi}{6}\right)$$

The cosine and the secant are even functions: $\cos(-\theta) = \cos \theta$ $\sec(-\theta) = \sec \theta$

The rest are odd functions: $\sin(-\theta) = -\sin \theta$ $\tan(-\theta) = -\tan \theta$
 $\csc(-\theta) = -\csc \theta$ $\cot(-\theta) = -\cot \theta$

Example 3 Find the exact value of: a) $\sin(-45^\circ)$ b) $\cos(-\pi)$ c) $\cot\left(-\frac{3\pi}{2}\right)$ d) $\tan\left(-\frac{37\pi}{4}\right)$

1.8 Circle Trigonometry Homework

For #21-36 find the exact value. Do not use a calculator. 21. $\sin 405^\circ$ 23. $\tan 405^\circ$
 25. $\cos 450^\circ$ 27. $\cot 390^\circ$ 29. $\cos \frac{33\pi}{4}$ 31. $\tan(21\pi)$ 33. $\sec \frac{17\pi}{4}$ 35. $\tan \frac{19\pi}{6}$
 For #37-54, find the exact value of each expression. Do not use a calculator.

37. $\sin(-60^\circ)$ 39. $\tan(-30^\circ)$ 41. $\sec(-60^\circ)$ 43. $\sin(-90^\circ)$
 45. $\tan\left(-\frac{\pi}{4}\right)$ 47. $\cos\left(-\frac{\pi}{4}\right)$ 49. $\tan(-\pi)$ 51. $\csc\left(-\frac{\pi}{4}\right)$ 53. $\sec\left(-\frac{\pi}{6}\right)$

In Problems 55-60, find the exact value of each expression. Do not use a calculator.

55. $\sin(-\pi) + \cos(5\pi)$ 57. $\sec(-\pi) + \csc\left(-\frac{\pi}{2}\right)$ 59. $\sin\left(-\frac{9\pi}{4}\right) - \tan\left(-\frac{9\pi}{4}\right)$

61. What is the domain of the sine function?
63. For what numbers θ is $f(\theta) = \tan \theta$ not defined?
65. For what numbers θ is $f(\theta) = \sec \theta$ not defined?
67. What is the range of the sine function?
69. What is the range of the tangent function?
71. What is the range of the secant function?
73. Is the sine function even, odd, or neither? Is its graph symmetric? With respect to what?
75. Is the tangent function even, odd, or neither? Is its graph symmetric? With respect to what?
77. Is the secant function even, odd, or neither? Is its graph symmetric? With respect to what?
79. If $\sin \theta = 0.3$, find the value of: $\sin \theta + \sin(\theta + 2\pi) + \sin(\theta + 4\pi)$.
81. If $\tan \theta = 3$, find the value of: $\tan \theta + \tan(\theta + \pi) + \tan(\theta + 2\pi)$.

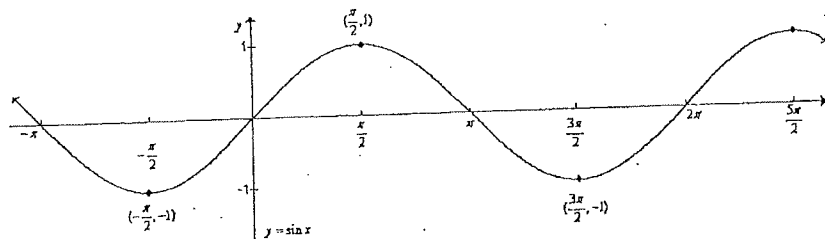
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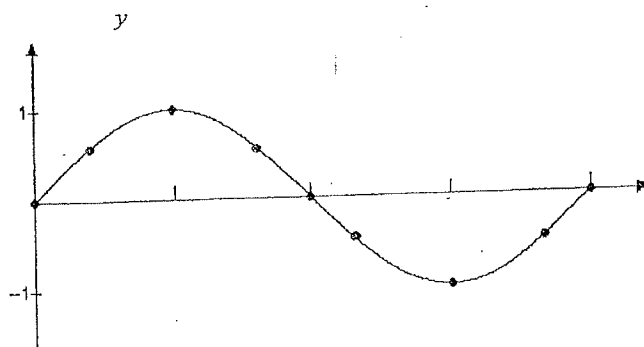
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1.9 Graphs & Transformations of the Sine and Cosine**Graph of the Sine Function**

$$y = \sin x, \quad -\infty \leq x \leq +\infty$$



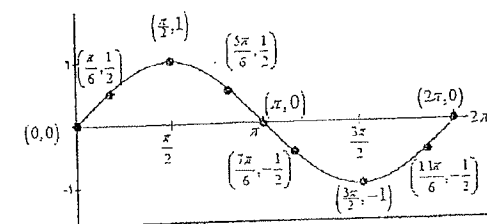
$$y = \sin x, \quad 0 \leq x \leq 2\pi$$



angle "x" sin x

0

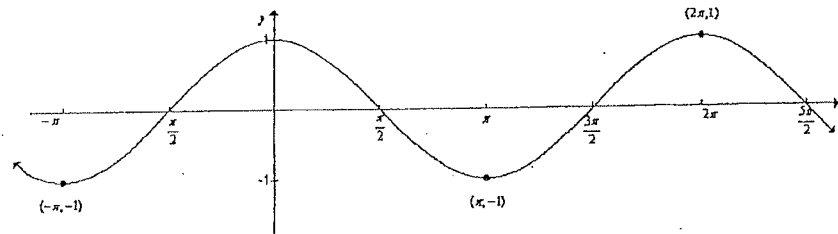
$$y = \sin x, \quad 0 \leq x \leq 2\pi$$



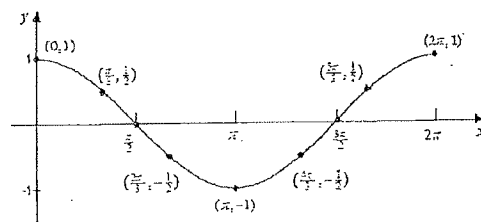
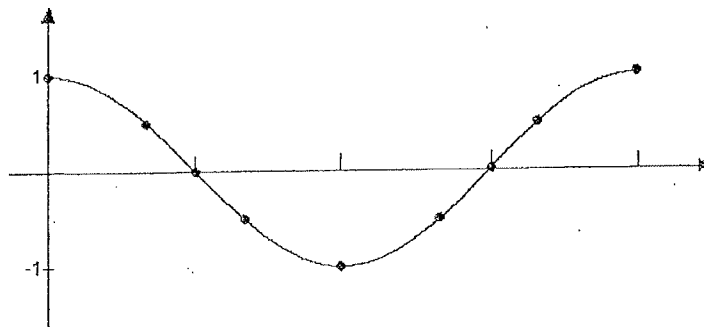
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Graphs of the Cosine Function

$$y = \cos x, 0 \leq x \leq 2\pi$$



angle "x"	$\cos x$
0	

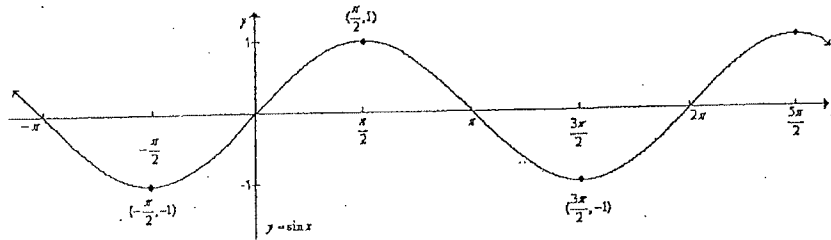
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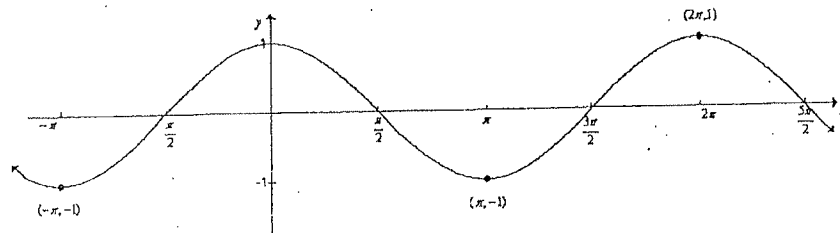
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Horizontal shifting of the graph.

EXAMPLE 1 Sine shifting: Use the graph of $y = \sin x$ to graph $y = \sin\left(x - \frac{\pi}{4}\right)$.

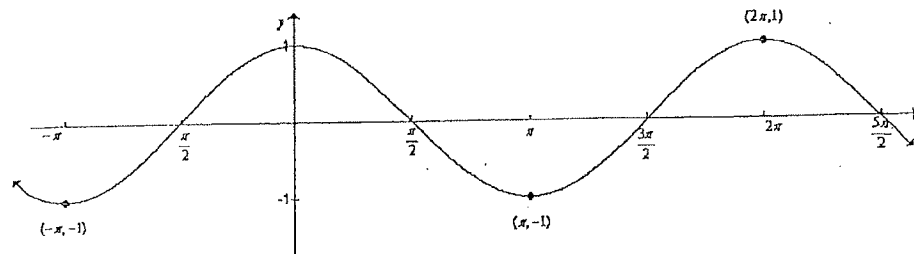


Example 3 Use the Graph of $y = \cos x$ to graph $y = 2\cos x$

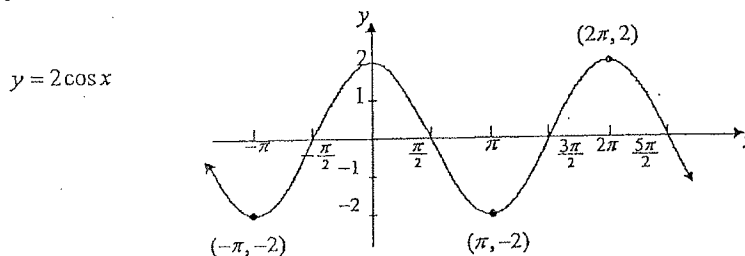


"1" is the amplitude (i.e. the largest vertical value) of the sine and cosine functions.
 " 2π " is the period of the sine and cosine functions.

Example Use the Graph of $y = \cos x$ to graph $y = \frac{1}{2} \cos x$.



In example 3 we obtained the graph of $y = 2 \cos x$, which is reproduced below. Notice that the y -values of $y = 2 \cos x$ lie between -2 and 2 , inclusive.



Theorem If $\omega > 0$, the amplitude and period of $y = A \sin(\omega x)$ and $y = A \cos(\omega x)$ are given by

$$\text{Amplitude} = |A|$$

$$\text{Sine \& Cosine Period} = \frac{2\pi}{\omega}$$

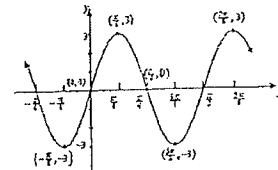
$$y = A \sin(\omega x - \varphi)$$

φ is the phase shift $|A|$ is amplitude ωx is the period

Example 5 Determine the amplitude and period of $y = 3 \sin(4x)$.

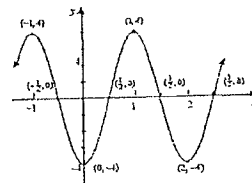
Solution: Amplitude $= |A| = 3$ and the Period $= \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

Example 6 Graph $y = 3\sin(4x)$



Solution:

Example 7 Determine the amplitude and period of $y = -4\cos(\pi x)$, and graph the function.



$y = A \sin(\omega x - \phi)$ ϕ is the *phase shift*
 $|A|$ is *amplitude* ωx is the *period*

1.9 Graphs & Transformations of the Sine and Cosine Homework

In Problems 21-35, use transformations to graph each function.

21. $y = 3\sin x$ 23. $y = -\cos x$ 25. $y = \sin x - 1$ 27. $y = \sin(x - \pi)$
 29. $y = \sin(\pi x)$ 31. $y = 2\sin x + 2$ 33. $y = 4\cos(2x)$ 35. $y = -2\sin x + 2$

In Problems 37-45, determine the amplitude and period of each function without graphing.

37. $y = 2\sin x$ 39. $y = -4\cos(2x)$ 41. $y = 6\sin(\pi x)$ 43. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$
 45. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$

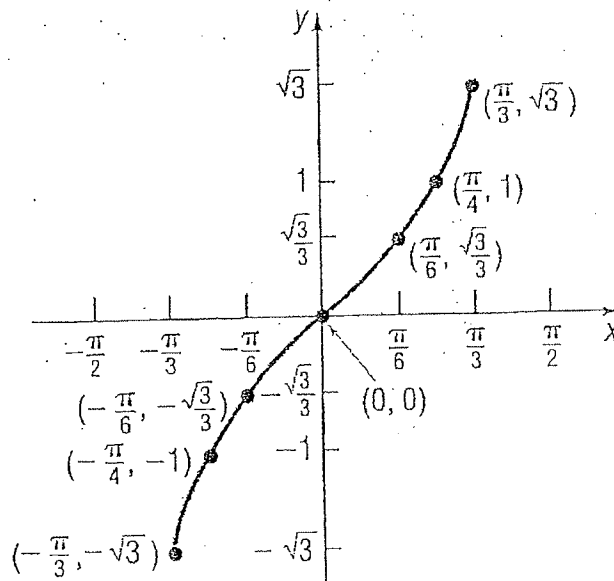
In Problems 61-69, graph each sinusoidal function.

61. $y = 5\sin(4x)$ 63. $y = 5\cos(\pi x)$ 65. $y = -2\cos(2\pi x) + 1$
 67. $y = -4\sin\left(\frac{1}{2}x\right)$ 69. $y = \frac{3}{2}\sin\left(-\frac{2}{3}x\right)$

1.10 Graphing the Tangent, Cotangent, Cosecant, SecantGraph $y = \tan x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

x	$y = \tan x$	(x, y)
$-\frac{\pi}{3}$	$-\sqrt{3} \approx -1.73$	$(-\frac{\pi}{3}, -1.73)$
$-\frac{\pi}{4}$	-1	$(-\frac{\pi}{4}, -1)$
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \approx -0.58$	$(-\frac{\pi}{6}, -0.58)$
0	0	(0,0)
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.58$	$(\frac{\pi}{6}, .58)$
$\frac{\pi}{4}$	1	$(\frac{\pi}{4}, 1)$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.73$	$(\frac{\pi}{3}, 1.73)$

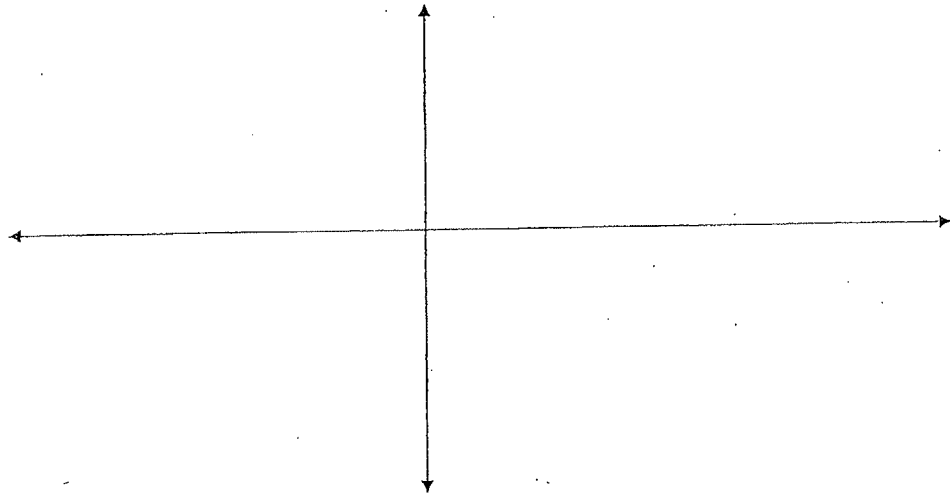
$$y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$



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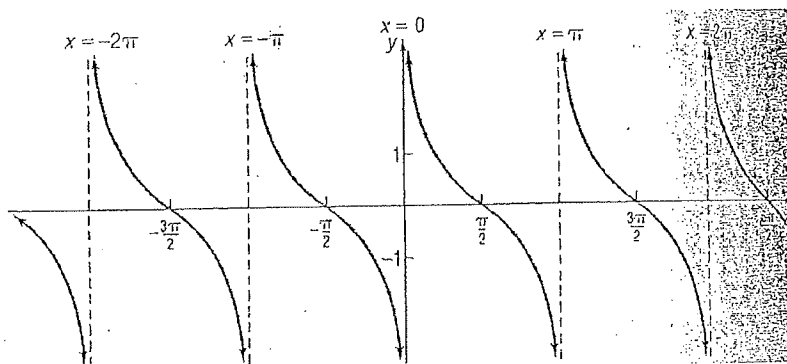
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Graph $y = \tan x$, $x = (-\infty, +\infty)$ 

The tangent function is discontinuous @ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots, \frac{(2n-1)\pi}{2}$
 $x = -\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, -\frac{7\pi}{2}, \dots, -\frac{(2n-1)\pi}{2}$

natural # $n = 1, 2, 3, \dots$ $2n = \text{an even number}$ $2n-1 = \text{an odd number}$

$$y = \cot x$$



Domain =

Range =

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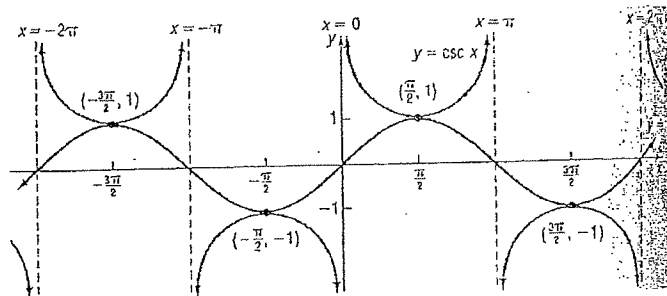
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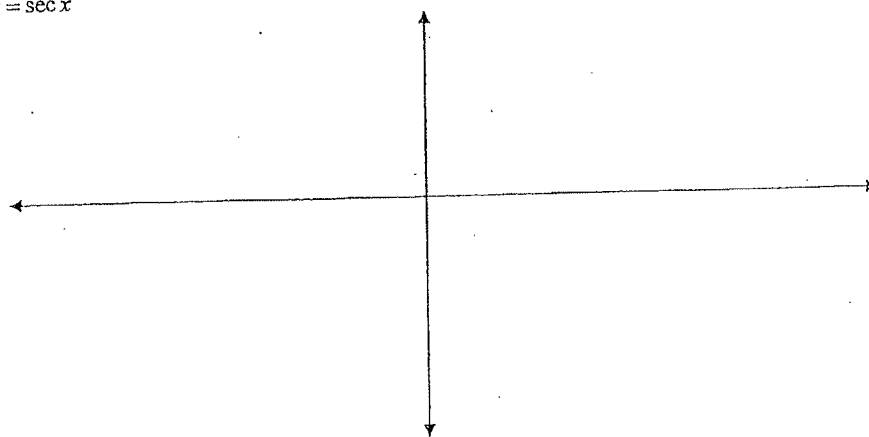
$y = \csc x$

Domain =

Range =



$y = \sec x$



Domain =

Range =

1.10 Graphing the Tangent, Cotangent, Cosecant, Secant

3. The graph of $y = \tan x$ is symmetric with respect to the _____ and has vertical asymptotes at _____.
5. It is easiest to graph $y = \sec x$ by first sketching the graph of _____.
7. What is the y -intercept of $y = \tan x$?
9. What is the y -intercept of $y = \sec x$?
11. For what x -values, $-2\pi \leq x \leq 2\pi$, does the $\sec x = 1$? For what values of x does $\sec x = -1$?
13. For what x -values, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \sec x$ have vertical asymptotes?
15. For what x -values, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \tan x$ have vertical asymptotes?

In Problems 21-44, graph each function.

21. $y = -\sec x$ 23. $y = \sec\left(x - \frac{\pi}{2}\right)$ 25. $y = \tan(x - \pi)$ 27. $y = 3 \tan(2x)$

29. $y = \sec(2x)$ 31. $y = \cot(\pi x)$ 33. $y = -3 \tan(4x)$ 35. $y = 2 \sec\left(\frac{1}{2}x\right)$

39. $y = \frac{1}{2} \cot\left(x - \frac{\pi}{4}\right)$ 41. $y = \tan x + 2$ 43. $y = \sec\left(x + \frac{\pi}{2}\right) - 1$

1.11 The Inverse Trig Functions

The Inverse Sine, Cosine, and Tangent Functions

When the angle is known, you can find the value of the trig function.

Find the sine ratio when the angle is $\frac{\pi}{3}$:

$$\sin \frac{\pi}{3} =$$

When the angle is not known, you must do the inverse.

Find the angle when the sine ratio is $\frac{1}{2}$.

$\sin \theta = \frac{1}{2}$ θ is the angle whose sine ratio is $\frac{1}{2}$.

In math symbols this looks like $\theta = \arcsin \frac{1}{2}$ or $\theta = \sin^{-1} \left(\frac{1}{2} \right)$

Example 1 Find the value of: $\sin^{-1} 1$

Example 2 Find the value of: a) $\sin^{-1}\left(-\frac{1}{2}\right)$ b) $\cos^{-1} 0$ c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Solution a) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ b) $\cos^{-1} 0 = \frac{\pi}{2}$ c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Example 3 Find an approximate value of: (a) $\sin^{-1}\frac{1}{3}$ (b) $\sin^{-1}\left(-\frac{1}{4}\right)$

Express the answer in radians rounded to two decimal places. Use a calculator.

Solution (a) $\sin^{-1}\frac{1}{3} \approx 0.34$ (b) $\sin^{-1}\left(-\frac{1}{4}\right) \approx -0.25$

Example 6 Find the exact value of: (a) $\cos^{-1}\left[\cos\left(\frac{\pi}{12}\right)\right]$ (b) $\cos[\cos^{-1}(-0.4)]$

Solution (a) $\cos^{-1}\left[\cos\left(\frac{\pi}{12}\right)\right] = \frac{\pi}{12}$ (b) $\cos[\cos^{-1}(-0.4)] = -0.4$

Example 7 Find the exact value of: a) $\tan^{-1}1$ b) $\tan^{-1}(-\sqrt{3})$

Example 9 Solve the equation $3\sin^{-1}x = \pi$

The solution set is $\left\{\frac{\sqrt{3}}{2}\right\}$

Graphs of Inverse Trig FunctionsThe Inverse Sine Function $y = \sin^{-1} x$, $x = [-1, +1]$ and $\theta = y = [-\frac{\pi}{2}, +\frac{\pi}{2}]$

$$\theta = x = \sin y$$

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \quad \text{when } x = [-\frac{\pi}{2}, +\frac{\pi}{2}]$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \quad \text{when } x = [-1, +1]$$

The Inverse Cosine Function $y = \cos^{-1} x$, $x = [-1, +1]$, $\theta = y = [0, \pi]$

$$\theta = x = \cos y$$

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \quad \text{when } x = [0, \pi]$$

$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \quad \text{when } x = [-1, +1]$$

The Inverse Tangent Function $y = \tan^{-1} x$, $x = (-\infty, +\infty)$, $\theta = y = (-\frac{\pi}{2}, +\frac{\pi}{2})$

$$\theta = x = \tan y$$

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad \text{when } x = [-\frac{\pi}{2}, +\frac{\pi}{2}]$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \quad \text{when } x = (-\infty, +\infty)$$

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \quad \text{when } x = [-\frac{\pi}{2}, +\frac{\pi}{2}]$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \quad \text{when } x = [-1, +1]$$

$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \quad \text{when } x = [0, \pi]$$

$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \quad \text{when } x = [-1, +1]$$

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad \text{when } x = [-\frac{\pi}{2}, +\frac{\pi}{2}]$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \quad \text{when } x = (-\infty, +\infty)$$

The Inverse Cosecant Function $y = \csc^{-1} x$, $x \leq -1$ or $x \geq +1$, and $y = [-\frac{\pi}{2}, +\frac{\pi}{2}]$, $y \neq 0$

$$\theta = x = \csc y$$

The Inverse Secant Function $y = \sec^{-1} x$ $x \leq -1$ or $x \geq +1$, and $\theta = y = [0, \pi]$, $y \neq \frac{\pi}{2}$

$$\theta = x = \sec y$$

The Inverse Cotangent Function $y = \cot^{-1} x$ $x = (-\infty, +\infty)$ and $\theta = y = (0, \pi)$

$$\theta = x = \cot y$$

Summary of Domain and Range for Inverse Trig Functions

$y = \sin^{-1} x$	$x = [-1, +1]$	and	$\theta = y = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$x = [-1, +1]$	and	$\theta = y = [0, \pi]$
$y = \tan^{-1} x$	$x = (-\infty, +\infty)$	and	$\theta = y = \left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$
$y = \sec^{-1} x$,	$x \leq -1$ or $x \geq 1$	and	$\theta = y = [0, \pi]$, $y \neq \frac{\pi}{2}$
$y = \csc^{-1} x$	$x \leq -1$ or $x \geq +1$	and	$\theta = y = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$, $y \neq 0$
$y = \cot^{-1} x$	$x = (-\infty, +\infty)$	and	$\theta = y = (0, \pi)$

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1.11 The Inverse Sine, Cosine, and Tangent Functions Homework

1. What is the Domain and the range of $y = \sin x$?
 2. A restriction on the domain of $f(x) = (x-1)^2$ to make it one-to-one would be $x =$ _____.
 3. If the domain of a one to one function is $[3, \infty)$, the range of its inverse is _____.
 4. *True or False:* The graph of $y = \cos x$ is decreasing on the interval $[0, \pi]$.
 5. $\tan \frac{\pi}{4} =$ _____; $\sin \frac{\pi}{3} =$ _____.
 6. $\sin\left(-\frac{\pi}{6}\right) =$ _____; $\cos \pi =$ _____.
 7. $y = \sin^{-1} x$ means _____, where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
 8. The value of $\sin^{-1}\left[\sin \frac{\pi}{2}\right]$ is _____.
 9. $\cos^{-1}\left[\cos \frac{\pi}{5}\right] =$ _____.
 10. *True or False:* The domain of $y = \sin^{-1} x$ is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
 11. *True or False:* $\sin(\sin^{-1} 0) = 0$ and $\cos(\cos^{-1} 0) = 0$.
 12. *True or False:* $y = \tan^{-1} x$ means $x = \tan y$, where $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
- In problems 13 – 23, find the exact value of each expression.
13. $\sin^{-1} 0$ 15. $\sin^{-1}(-1)$ 17. $\tan^{-1} 0$ 19. $\sin^{-1} \frac{\sqrt{2}}{2}$ 21. $\tan^{-1} \sqrt{3}$ 23. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
- For # 25 – 35 use a calculator to find the value rounded to two decimal places.
25. $\sin^{-1} 0.1$ 27. $\tan^{-1} 5$ 29. $\cos^{-1} \frac{7}{8}$ 31. $\tan^{-1}(-0.4)$ 33. $\sin^{-1}(-0.12)$ 35. $\cos^{-1} \frac{\sqrt{2}}{3}$
- In problems 37 – 43 find the exact value of each expression. Do not use a calculator.
37. $\sin[\sin^{-1}(0.54)]$ 39. $\cos^{-1}\left[\cos \frac{4\pi}{5}\right]$ 41. $\tan[\tan^{-1}(-3.5)]$ 43. $\sin^{-1}\left[\sin\left(-\frac{3\pi}{7}\right)\right]$
- In problems 45 – 55, do not use a calculator. For your answers, also say why or why not.
45. Does $\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$?
 47. Does $\sin[\sin^{-1}(2)] = 2$?
 49. Does $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$?
 51. Does $\cos\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$?
 53. Does $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$?
 55. Does $\tan[\tan^{-1}(2)] = 2$?
- In problems 57 – 63, find the exact solution of each equation.
57. $4 \sin^{-1}(x) = \pi$ 59. $3 \cos^{-1}(2x) = 2\pi$ 61. $3 \tan^{-1} x = \pi$ 63. $4 \cos^{-1} x - 2\pi = 2 \cos^{-1} x$

1.12 Values of Inverse Trig Functions

Find the exact value of $\sin^{-1}(\sin \frac{5\pi}{4})$.

Find the exact value of $\sin(\tan^{-1} \frac{1}{2})$.

Find the exact value of $\cos(\sin^{-1} \frac{1}{2})$

Find the exact value of $\tan(\cos^{-1}(-\frac{1}{3}))$

Use your calculator to approximate the value in radians to two decimal places:

a) $\sec^{-1}3$

b) $\csc^{-1}(-4)$

c) $\cot^{-1}(\frac{1}{2})$

d) $\cot^{-1}(-2)$

1.12 Values of Inverse Trig Functions Homework

- What is the domain and the range of $y = \sec x$?
- Is the graph of $y = \sec x$ increasing on the intervals $\left[0, \frac{\pi}{2}\right)$ and $\left[\frac{\pi}{2}, \pi\right)$?
- If $\cot \theta = -2$ and $0 < \theta < \pi$, then $\cos \theta =$ _____.
- $y = \sec^{-1} x$ means _____, where $|x|$ _____ and _____ $\leq y \leq$ _____, $y \neq \frac{\pi}{2}$.
- $\cos(\tan^{-1} 1) =$ _____.
- True or false: You cannot obtain exact values for the inverse secant function.
- True or false: $\csc^{-1} 0.5$ is not defined.
- True or false: The domain of the inverse cotangent function is the set of real numbers.

In problems 9 – 35, find the value of each expression.

- | | | |
|--|--|---|
| 9. $\cos\left(\sin^{-1} \frac{\sqrt{2}}{2}\right)$ | 11. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ | 13. $\sec\left(\cos^{-1} \frac{1}{2}\right)$ |
| 15. $\csc(\tan^{-1} 1)$ | 17. $\sin[\tan^{-1}(-1)]$ | 19. $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ |
| 21. $\cos^{-1}\left(\cos \frac{5\pi}{4}\right)$ | 23. $\sin^{-1}\left[\sin\left(-\frac{7\pi}{6}\right)\right]$ | 25. $\tan\left(\sin^{-1} \frac{1}{3}\right)$ |
| 27. $\sec\left(\tan^{-1} \frac{1}{2}\right)$ | 29. $\cot\left[\sin^{-1}\left(-\frac{\sqrt{2}}{3}\right)\right]$ | 31. $\sin[\tan^{-1}(-3)]$ |
| 33. $\sec\left(\sin^{-1} \frac{2\sqrt{5}}{5}\right)$ | 35. $\sin^{-1}\left(\cos \frac{3\pi}{4}\right)$ | |

In problems 37 – 43, find the exact value of each expression.

- | | | | |
|--------------------------|---------------------|-------------------------------------|---|
| 37. $\cot^{-1} \sqrt{3}$ | 39. $\csc^{-1}(-1)$ | 41. $\sec^{-1} \frac{2\sqrt{3}}{3}$ | 43. $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ |
|--------------------------|---------------------|-------------------------------------|---|

Use a calculator to find the value of each expression rounded to two decimal places.

- | | | | |
|--|--|---------------------|----------------------------|
| 45. $\sec^{-1} 4$ | 47. $\cot^{-1} 2$ | 49. $\csc^{-1}(-3)$ | 51. $\cot^{-1}(-\sqrt{5})$ |
| 53. $\csc^{-1}\left(-\frac{3}{2}\right)$ | 55. $\cot^{-1}\left(-\frac{3}{2}\right)$ | | |

$f(x) = \sin x$, and $h(x) = \tan x$. Find the exact value of each composite function.

- | | | | |
|--|---|---|---|
| 57. $g\left(f^{-1}\left(\frac{12}{13}\right)\right)$ | 59. $g^{-1}\left(f\left(\frac{7\pi}{4}\right)\right)$ | 61. $h\left(f^{-1}\left(-\frac{3}{5}\right)\right)$ | 63. $g\left(h^{-1}\left(\frac{12}{5}\right)\right)$ |
| 65. $g^{-1}\left(f\left(-\frac{4\pi}{3}\right)\right)$ | 67. $h\left(g^{-1}\left(-\frac{1}{4}\right)\right)$ | | |

1.13 Trigonometric Identities

Identities and proofs, addition formulas, double-angle, half-angle, reduction formulas
Methods used to Simplify Trigonometric Expressions

Example 1

- a) Simplify $\frac{\cot \theta}{\csc \theta}$ by rewriting in terms of sine and cosine functions.
- b) Show that $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$ by multiplying the numerator & denominator by conjugate of the denominator i.e. $1 - \sin \theta$.
- c) Simplify $\frac{1 + \sin \theta}{\sin \theta} + \frac{\cot \theta - \cos \theta}{\cos \theta}$ by adding two fractions i.e. getting a common denominator
- d) Simplify $\frac{\sin^2 \theta - 1}{\tan \theta \sin \theta - \tan \theta}$ by factoring.

a) $\frac{\cot \theta}{\csc \theta}$

Solution: a) $\frac{\cot \theta}{\csc \theta} = \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1} = \cos \theta$

b) $\frac{\cos \theta}{1 + \sin \theta} \Rightarrow \frac{\cos \theta}{1 + \sin \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta} \Rightarrow \frac{\cos \theta(1 - \sin \theta)}{1 - \sin^2 \theta} \Rightarrow \frac{\cos \theta(1 - \sin \theta)}{\cos^2 \theta} \Rightarrow \frac{1 - \sin \theta}{\cos \theta}$

c) $\frac{1 + \sin \theta}{\sin \theta} + \frac{\cot \theta - \cos \theta}{\cos \theta} \Rightarrow \frac{1 + \sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} + \frac{\cot \theta - \cos \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} \Rightarrow \frac{\cos \theta + \sin \theta \cos \theta + \cot \theta \sin \theta}{\sin \theta \cos \theta}$
 $\Rightarrow \frac{\cos \theta + \frac{\cos \theta}{\sin \theta} \cdot \sin \theta}{\sin \theta \cos \theta} \Rightarrow \frac{\cos \theta + \cos \theta}{\sin \theta \cos \theta} \Rightarrow \frac{2 \cos \theta}{\sin \theta \cos \theta} \Rightarrow \frac{2}{\sin \theta}$

d) $\frac{\sin^2 \theta - 1}{\tan \theta \sin \theta - \tan \theta} \Rightarrow \frac{(\sin \theta + 1)(\sin \theta - 1)}{\tan \theta (\sin \theta - 1)} \Rightarrow \frac{\sin \theta + 1}{\tan \theta}$

Example 2 Verify that $\csc \theta \cdot \tan \theta = \sec \theta$

Hint: Start with the left side, because it contains the more complicated expression.

Example 3 Verify that $\sin^2(-\theta) + \cos^2(-\theta) = 1$

Example 4 Verify $\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos \theta - \sin \theta$

Example 5 Verify that $\frac{1 + \tan \theta}{1 + \cot \theta} = \tan \theta$

Example 6 Verify that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$

Example 7 Verify that $\frac{\tan \theta + \cot \theta}{\sec \theta \csc \theta} = 1$

Example 8 Verify that $\frac{1 - \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 + \sin \theta}$

Guidelines for Establishing Identities

1. It is usually best to start with the more complicated expression.
2. Add or subtract fractions.
3. Rewrite one side in terms of sine and cosine functions only.
4. Always keep in mind the form of the expression on the other side.

1.13 Trigonometric Identities Homework

1. True or False: $\sin^2 \theta = 1 - \cos^2 \theta$
2. True or False: $\sin(-\theta) + \cos(-\theta) = \cos \theta - \sin \theta$.
4. $\tan^2 \theta - \sec^2 \theta$.
5. $\cos(-\theta) - \cos \theta =$ _____.
6. True or False: $\sin(-\theta) + \sin \theta = 0$ for any value of θ .

Simplify problems 9 – 17 by following the indicated direction.

9. Rewrite in terms of sine and cosine functions:
- $\tan \theta \csc \theta$

11. Multiply $\frac{\cos \theta}{1 - \sin \theta}$ by $\frac{1 + \sin \theta}{1 + \sin \theta}$

13. Rewrite over a common denominator: $\frac{\sin \theta + \cos \theta}{\cos \theta} + \frac{\cos \theta - \sin \theta}{\sin \theta}$

15. Multiply and simplify: $\frac{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta) - 1}{\sin \theta \cos \theta}$

17. Factor and simplify: $\frac{3 \sin^2 \theta + 4 \sin \theta + 1}{\sin^2 \theta + 2 \sin \theta + 1}$

In problems 19 – 103, establish each identity.

19. $\csc \theta \cos \theta = \cot \theta$

21. $1 + \tan^2(-\theta) = \sec^2 \theta$

23. $\cos \theta (\tan \theta + \cot \theta) = \csc \theta$

25. $\tan \theta \cot \theta - \cos^2 \theta = \sin^2 \theta$

27. $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$

29. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

31. $\cos^2 \theta (1 + \tan^2 \theta) = 1$

33. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

35. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

37. $\sec \theta - \tan \theta = \frac{\cos \theta}{1 + \sin \theta}$

39. $3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$

41. $1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$

43. $\frac{1 + \tan \theta}{1 - \tan \theta} = \frac{\cot \theta + 1}{\cot \theta - 1}$

45. $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$

47. $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{\csc \theta + 1}{\csc \theta - 1}$

49. $\frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$

51. $\frac{\sin \theta}{\sin \theta - \cos \theta} = \frac{1}{1 - \cot \theta}$

53. $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$

$$55. \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta \quad 57. \tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$$

1.14 The Sum and Difference Formulas

Sum and Difference Formulas for the Cosine Function

The cosine of the *sum* of two angles equals the cosine of the first angle times the cosine of the second angle *minus* the sine of the first angle times the sine of the second angle.

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Memorize these:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Example 1 Find the exact value of $\cos 75^\circ$. Hint: $75 = 45 + 30$

Solution: $\frac{\sqrt{6} - \sqrt{2}}{4}$

Example 2 Find the exact value of $\cos \frac{\pi}{12}$. Hint: $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

Solution: $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

Verify the cofunction formula: $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ Cofunctions of complementary angles are equal.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos \frac{\pi}{2} \cos \theta + \sin \frac{\pi}{2} \sin \theta = 0 \cos \theta + 1 \sin \theta = \sin \theta$$

Verify the cofunction formula: $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

Using the previous cosine cofunction formula i.e. $\left\{ \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \right\}$ in reverse gives us

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{2} - \theta\right)\right] = \cos \theta \quad \text{thus establishing the cofunction formula.}$$

Sum and Difference Formulas for the Sine Function

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Using the cofunction formula for cosine in reverse gives us

$$\begin{aligned} \sin(\alpha + \beta) &= \cos\left(\frac{\pi}{2} - (\alpha + \beta)\right) = \cos\left(\left(\frac{\pi}{2} - \alpha\right) - \beta\right) \\ &= \cos\left(\frac{\pi}{2} - \alpha\right) \cos \beta + \sin\left(\frac{\pi}{2} - \alpha\right) \sin \beta \\ &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{aligned}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

Using the sum formula for sine gives us

$$\begin{aligned} \sin(\alpha - \beta) &= \sin[\alpha + (-\beta)] \\ &= \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta) \\ &= \sin \alpha \cos \beta + \cos \alpha (-\sin \beta) \end{aligned}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Example 3 Find the exact value of $\sin \frac{7\pi}{12}$.

$$\text{Solution} = \frac{1}{4}(\sqrt{2} + \sqrt{6})$$

Example 4 Find the exact value of $\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ$

$$\text{Solution} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

Example 5 If $\sin \alpha = \frac{4}{5}$, $\alpha = \left(\frac{\pi}{2}, \pi\right)$ and $\sin \beta = -\frac{2}{\sqrt{5}}$, $\beta = \left(\pi, \frac{3\pi}{2}\right)$
 find the exact value of (a) $\cos \alpha$ (b) $\cos \beta$

(c) $\cos(\alpha + \beta)$

(d) $\sin(\alpha + \beta)$

Solution: (a) $\cos \alpha = -\frac{3}{5}$ (b) $\cos \beta = -\frac{\sqrt{5}}{5}$ (c) $\cos(\alpha + \beta) = \frac{11\sqrt{5}}{25}$ (d) $\sin(\alpha + \beta) = \frac{2\sqrt{5}}{25}$

Example 6 Verify that $\frac{\cos(\alpha - \beta)}{\sin \alpha \sin \beta} = \cot \alpha \cot \beta + 1$

Example 7 Prove that $\tan(\theta + \pi) = \tan \theta$. This verifies that the tangent has a period of π .

Example 8 Prove the identity: $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot \theta$

Proof of the sum and difference formulas for the tangent function

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Proof:

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$

divide the numerator and the denominator by $\cos \alpha \cos \beta$

$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{\frac{\cos \beta}{\cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad \text{Q.E.D.}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Proof:

$$\tan(\alpha - \beta) = \tan[\alpha + (-\beta)]$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan(\beta)}{1 - \tan \alpha (-\tan \beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad \text{Q.E.D.}$$

1.14 The Sum and Difference Formulas Homework

1. The distance d from the point $(2, -3)$ to the point $(5, 1)$ is _____.

2. If $\sin \theta = \frac{4}{5}$ and θ is in quadrant II, then $\cos \theta =$ _____.

3. (a) $\sin \frac{\pi}{4} \cos \frac{\pi}{3} =$ _____ (b) $\tan \frac{\pi}{4} - \sin \frac{\pi}{6} =$ _____.

4. $\cos(\alpha + \beta) = \cos \alpha \cos \beta$ _____ $\sin \alpha \sin \beta$

5. $\sin(\alpha - \beta) = \sin \alpha \cos \beta$ _____ $\cos \alpha \sin \beta$

6. True or False: $\sin(\alpha + \beta) = \sin \alpha + \sin \beta + 2 \sin \alpha \sin \beta$

7. True or False: $\tan 75^\circ = \tan 30^\circ + \tan 45^\circ$

8. True or False: $\cos\left(\frac{\pi}{2} - \theta\right) = \cos \theta$

In problems 9 – 19, find the exact value of each expression.

9. $\sin \frac{5\pi}{12}$ 11. $\cos \frac{7\pi}{12}$ 13. $\cos 165^\circ$ 15. $\tan 15^\circ$ 17. $\sin \frac{17\pi}{12}$ 19. $\sec\left(-\frac{\pi}{12}\right)$

In problems 21 – 29, find the exact value of each expression.

21. $\sin 20^\circ \cos 10^\circ + \cos 20^\circ \sin 10^\circ$ 23. $\cos 70^\circ \cos 20^\circ - \sin 70^\circ \sin 20^\circ$

25. $\frac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \tan 25^\circ}$ 27. $\sin \frac{\pi}{12} \cos \frac{7\pi}{12} - \cos \frac{\pi}{12} \sin \frac{7\pi}{12}$ 29. $\cos \frac{\pi}{12} \cos \frac{5\pi}{12} + \sin \frac{5\pi}{12} \sin \frac{\pi}{12}$

Find the exact value of problems 31–35 under the given conditions:

(a) $\sin(\alpha + \beta)$ (b) $\cos(\alpha + \beta)$ (c) $\sin(\alpha - \beta)$ (d) $\tan(\alpha - \beta)$

31. $\sin \alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$; $\cos \beta = \frac{2\sqrt{5}}{5}$, $-\frac{\pi}{2} < \beta < 0$

33. $\tan \alpha = -\frac{4}{3}$, $\frac{\pi}{2} < \alpha < \pi$; $\cos \beta = \frac{1}{2}$, $0 < \beta < \frac{\pi}{2}$

35. $\sin \alpha = \frac{5}{13}$, $-\frac{3\pi}{2} < \alpha < -\pi$; $\tan \beta = -\sqrt{3}$, $\frac{\pi}{2} < \beta < \pi$

37. If $\sin \theta = \frac{1}{3}$, θ is in quadrant 2, find the exact value of:

(a) $\cos \theta$ (b) $\sin\left(\theta + \frac{\pi}{6}\right)$ (c) $\cos\left(\theta - \frac{\pi}{3}\right)$ (d) $\tan\left(\theta + \frac{\pi}{4}\right)$

In problems 45–55, establish each identity.

45. $\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$

47. $\sin(\pi - \theta) = \sin \theta$ 49. $\sin(\pi + \theta) = -\sin \theta$ 51. $\tan(\pi - \theta) = -\tan \theta$

53. $\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos \theta$ 55. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$

In problems 71 & 73, find the exact value of each expression.

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71. $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}0\right)$

73. $\sin\left[\sin^{-1}\frac{3}{5} - \cos^{-1}\left(-\frac{4}{5}\right)\right]$

1.15 Double-angle and Power Reducing Formulas**Double-angle Formulas**

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power Reducing Formulas

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Derive the formula for $\sin(2\theta)$ Derive the formulas for $\cos(2\theta)$

Derive the formula for $\tan(2\theta)$

If $\sin \theta = \frac{3}{5}$, $\theta = \left(\frac{\pi}{2}, \pi\right)$, find

a) $\sin(2\theta)$

b) $\cos(2\theta)$

Derive the formula for $\sin^2 \theta$

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Derive the formula for $\cos^2 \theta$

Derive the formula for $\tan^2 \theta$

Rewrite $\cos^4 \theta$ without any powers of sine or cosine greater than 1.

1.15 Double-angle and Power-reducing Formulas Homework

1. $\cos(2\theta) = \cos^2 \theta - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} - 1 = 1 - \underline{\hspace{1cm}}.$

2. $\sin^2 \frac{\theta}{2} = \underline{\hspace{1cm}}.$

3. $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\underline{\hspace{1cm}}}.$

4. Does $\cos(2\theta)$ have these equivalent forms: $\cos^2 \theta - \sin^2 \theta$, $1 - 2\sin^2 \theta$, $2\cos^2 \theta - 1$?

5. Does $\sin(2\theta)$ have these equivalent forms: $2\sin \theta \cos \theta$ and $\sin^2 \theta - \cos^2 \theta$?

6. Does $\tan(2\theta) + \tan(2\theta) = \tan(4\theta)$?

In problems 7 - 17, $0 \leq \theta < 2\pi$. Use the information given to find the exact value of:

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$ (c) $\sin \frac{\theta}{2}$ (d) $\cos \frac{\theta}{2}$

7. $\sin \theta = \frac{3}{5}, 0 < \theta < \frac{\pi}{2}$

9. $\tan \theta = \frac{4}{3}, \pi < \theta < \frac{3\pi}{2}$

11. $\cos \theta = -\frac{\sqrt{6}}{3}, \frac{\pi}{2} < \theta < \pi$

13. $\sec \theta = 3, \sin \theta > 0$

15. $\cot \theta = -2, \sec \theta < 0$

17. $\tan \theta = -3, \sin \theta < 0$

Use Power Reducing Formulas to find the exact value of #19-27.

19. $\sin 22.5^\circ$ 21. $\tan \frac{7\pi}{8}$ 23. $\cos 165^\circ$ 25. $\sec \frac{15\pi}{8}$ 27. $\sin\left(-\frac{\pi}{8}\right)$

41. Show that $\sin^4 \theta = \frac{3}{8} - \frac{1}{2} \cos(2\theta) + \frac{1}{8} \cos(4\theta)$

In problems 47 - 67, establish each identity.

47. $\cos^4 \theta - \sin^4 \theta = \cos(2\theta)$ 49. $\cot(2\theta) = \frac{\cot^2 \theta - 1}{2 \cot \theta}$ 51. $\sec(2\theta) = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

53. $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$ 55. $\frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cot \theta - 1}{\cot \theta + 1}$ 57. $\sec^2 \frac{\theta}{2} = \frac{2}{1 + \cos \theta}$

59. $\cot^2 \frac{\theta}{2} = \frac{\sec \theta + 1}{\sec \theta - 1}$ 61. $\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$ 63. $\frac{\sin(3\theta)}{\sin \theta} - \frac{\cos(3\theta)}{\cos \theta} = 2$

65. $\tan(3\theta) = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$ 67. $\ln |\sin \theta| = \frac{1}{2} (\ln |1 - \cos(2\theta)| - \ln 2)$

In problems 69 - 79, find the exact value of each expression.

69. $\sin\left(2 \sin^{-1} \frac{1}{2}\right)$ 71. $\cos\left(2 \sin^{-1} \frac{3}{5}\right)$ 73. $\tan\left[2 \cos^{-1}\left(-\frac{3}{5}\right)\right]$

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75. $\sin\left(2\cos^{-1}\frac{4}{5}\right)$

77. $\sin^2\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$

79. $\sec\left(2\tan^{-1}\frac{3}{4}\right)$

1.16 Solving Trigonometric Equations

Example 1: Is $\theta = \frac{\pi}{4}$ a solution to the equation $\sin\theta = \frac{1}{2}$? If not, what is the solution?

Example 2: Solve the equation $\cos\theta = \frac{1}{2}$.

Solution: $\theta = \frac{\pi}{3} + 2k\pi$, $\theta = \frac{5\pi}{3} + 2k\pi$ where k is an integer

Example 3: Solve the equation $2\sin\theta + \sqrt{3} = 0$, $0 \leq \theta < 2\pi$.

Hint: Solve for sine. Solution: $x = \frac{4\pi}{3}, \frac{5\pi}{3}$ or $240^\circ, 300^\circ$

Example 4: Solve the equation $\sin 2\theta = \frac{1}{2}$ where $\theta \in [0, 2\pi)$.

Solution: $\theta = \frac{\pi}{12} + k\pi, \theta = \frac{5\pi}{12} + k\pi, k = 0, 1, 2 \Rightarrow \theta = \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

Example 1 Solve the quadratic trinomial equation: $2\sin^2 \theta - 3\sin \theta + 1 = 0, 0 \leq \theta < 2\pi$

Hint: factor. The solution set is $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\right\}$.

Example 2 Solve the equation using identities: $3\cos\theta + 3 = 2\sin^2\theta$, $0 \leq \theta < 2\pi$

Hint: Use Pythagorean Identity $\sin^2\theta = \dots$ The solution set is $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$.

Example 3 Solve the equation: $\cos(2\theta) + 3 = 5\cos\theta$, $0 \leq \theta < 2\pi$

Hint: Reduce 2θ . Use double angle formula $\cos 2\theta = 2\cos^2\theta - 1$ The solution set is $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$.

Example 4 Solve the equation: $\cos^2\theta + \sin\theta = 2$, $0 \leq \theta < 2\pi$

Hint: Use Pythagorean Identity to write in terms of $\sin \theta$. Solution: $\sin^2 \theta - \sin \theta + 1 = 0$

1.16 Solving Trigonometric Equations

In # 7 - 29, solve each equation on the interval $0 \leq \theta < 2\pi$.

- | | | |
|-------------------------------------|---|---|
| 7. $2\sin \theta + 3 = 2$ | 9. $4\cos^2 \theta = 1$ | 11. $2\sin^2 \theta - 1 = 0$ |
| 13. $\sin(3\theta) = -1$ | 15. $\cos(2\theta) = -\frac{1}{2}$ | 17. $\sec \frac{3\theta}{2} = -2$ |
| 19. $2\sin \theta + 1 = 0$ | 21. $\tan \theta + 1 = 0$ | 23. $4\sec \theta + 6 = -2$ |
| 25. $3\sqrt{2}\cos \theta + 2 = -1$ | 27. $\cos\left(2\theta - \frac{\pi}{2}\right) = -1$ | 29. $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$ |

Solve #31 - 39. Give a general formula for all the solutions. List six solutions.

- | | | | | |
|---------------------------------|---|-----------------------|------------------------------------|---|
| 31. $\sin \theta = \frac{1}{2}$ | 33. $\tan \theta = -\frac{\sqrt{3}}{3}$ | 35. $\cos \theta = 0$ | 37. $\cos(2\theta) = -\frac{1}{2}$ | 39. $\sin \frac{\theta}{2} = -\frac{\sqrt{3}}{2}$ |
|---------------------------------|---|-----------------------|------------------------------------|---|

Solve #41 - 51 for $\theta = [0, 2\pi)$. Round answers to two decimal places.

- | | | | |
|----------------------------|----------------------------|--------------------------|------------------------|
| 41. $\sin \theta = 0.4$ | 43. $\tan \theta = 5$ | 45. $\cos \theta = -0.9$ | 47. $\sec \theta = -4$ |
| 49. $5\tan \theta + 9 = 0$ | 51. $3\sin \theta - 2 = 0$ | | |

- Find the real solutions of $4x^2 - x - 5 = 0$.
- Find the real solutions of $x^2 - x - 1 = 0$.
- Find the real solutions of $(2x - 1)^2 - 3(2x - 1) - 4 = 0$.
- Use a graphing utility to solve $5x^3 - 2 = x - x^2$. Round answers to two decimal places.

In problems 5 - 45, solve each equation on the interval $0 \leq \theta < 2\pi$.

- | | | |
|---|---|---|
| 5. $2\cos^2 \theta + \cos \theta = 0$ | 7. $2\sin^2 \theta - \sin \theta - 1 = 0$ | 9. $(\tan \theta - 1)(\sec \theta - 1) = 0$ |
| 11. $\sin^2 \theta - \cos^2 \theta = 1 + \cos \theta$ | 13. $\sin^2 \theta = 6(\cos \theta + 1)$ | 15. $\cos(2\theta) + 6\sin^2 \theta = 4$ |
| 17. $\cos \theta = \sin \theta$ | 19. $\tan \theta = 2\sin \theta$ | 21. $\sin \theta = \csc \theta$ |
| 23. $\cos(2\theta) = \cos \theta$ | 25. $\sin(2\theta) + \sin(4\theta) = 0$ | 27. $\cos(4\theta) - \cos(6\theta) = 0$ |
| 29. $1 + \sin \theta = 2\cos^2 \theta$ | 31. $2\sin^2 \theta - 5\sin \theta + 3 = 0$ | 33. $3(1 - \cos \theta) = \sin^2 \theta$ |
| 35. $\tan^2 \theta = \frac{3}{2}\sec \theta$ | 37. $3 - \sin \theta = \cos(2\theta)$ | 41. $\sin \theta - \sqrt{3}\cos \theta = 1$ |
| 43. $\tan(2\theta) + 2\sin \theta = 0$ | 45. $\sin \theta + \cos \theta = \sqrt{2}$ | |

In # 47 - 51, find the real zeros of each trig function on the interval $0 \leq x < 2\pi$.

- | | | |
|----------------------------|--------------------------------|---------------------------|
| 47. $f(x) = 4\cos^2 x - 1$ | 49. $f(x) = \sin(2x) - \sin x$ | 51. $\sin x + \cos x = x$ |
|----------------------------|--------------------------------|---------------------------|

Solve #53 - 63 with a calculator. Round solution(s) to two decimal places.

- | | | |
|-----------------------|--------------------------|---------------------------|
| 53. $x + 5\cos x = 0$ | 55. $22x - 17\sin x = 3$ | 57. $\sin x + \cos x = x$ |
|-----------------------|--------------------------|---------------------------|

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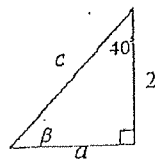
59. $x^2 = x + 3\cos(2x)$

61. $x^2 - 2\sin(2x) = 3x$

63. $6\sin x - e^x = 2, x > 0$

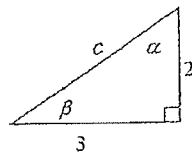
1.17 Applications Involving Right Triangles

Example 1: If $b=2$ and $\alpha = 40^\circ$, find a, c , and β in the right triangle.



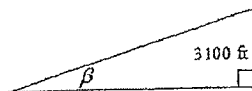
Solution: $a = 2 \tan 40^\circ \approx 1.68$ and $c = \frac{2}{\cos 40^\circ} \approx 2.61$, $\beta = 50^\circ$.

Example 2: If $a=3$ and $b=2$, find c, α , and β .



Solution: $c = \sqrt{13} \approx 3.61$, $\alpha = \tan^{-1} \frac{3}{2}$, $\beta = 33.7^\circ$

Example 3: A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination (grade) of the trail? That is, what is the angle β in Figure 4?



Solution: $\beta = \sin^{-1} \frac{3100}{14,100} \approx 12.7^\circ$ The inclination (grade) of the trail is approximately 12.7°

1.17 Applications Involving Right Triangles Homework

1. In a right triangle, if the length of the hypotenuse is 5 and the length of one of the other sides is 3, what is the length of the third side?
2. *True or False:* The angles 52° and 48° are complementary.
3. If θ is an acute, solve the equation $\tan \theta = \frac{1}{2}$. Round answer to one decimal place.
4. If θ is an acute angle, solve the equation $\sin \theta = \frac{1}{2}$.
5. *True or False:* In a right triangle, one of the angles is 90° and the sum of the other two angles is 90° .
6. In navigation or surveying, the _____ from a point O to a point P equals the acute angle θ between ray OP and the vertical line through O , the north – south line.
7. *True or False:* In a right triangle, if two sides are known, we can solve the triangle.
8. *True or False:* In a right triangle, if we know the two acute angles, we can solve the triangle.

In problems 9 – 21, using the given information, solve the right triangle.

9. $b = 5$, $\beta = 20^\circ$; find a, c , and α
11. $a = 6$, $\beta = 40^\circ$; find b, c , and α
13. $b = 4$, $\alpha = 10^\circ$; find a, c , and β
15. $a = 5$, $\alpha = 25^\circ$; find b, c , and β
17. $c = 9$, $\beta = 20^\circ$; find a, c , and α
19. $a = 5$, $b = 3$; find c, α , and β
21. $a = 2$, $c = 5$; find b, α , and β
23. The hypotenuse of a right triangle is 5 inches. If one leg is 2 inches, find the degree measure of each angle.

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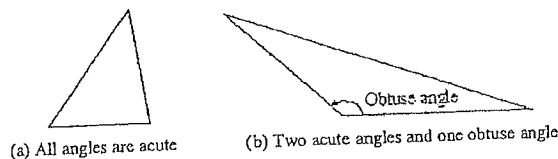
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24. The hypotenuse of a right triangle is 3 feet. If one leg is 1 foot, find the degree measure of each angle.
25. At 10 am on April 26, 2005, a building 300 feet high casts a shadow 50 feet long. What is the angle of elevation of the Sun?

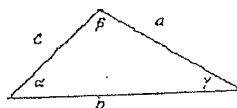
1.18 Law of Sines

If none of the angles of a triangle is a right angle, the triangle is called **oblique**.

An oblique triangle will have either three acute angles
or two acute angles and one obtuse angle (an angle between 90° and 180°).



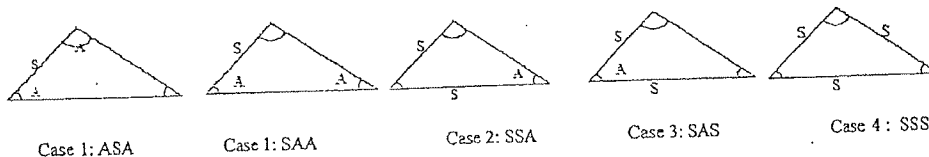
In the discussion that follows, we will always label an oblique triangle so that a is opposite angle α , side b is opposite angle β , and side c is opposite angle γ .



To solve an oblique triangle means to find the lengths of its sides and the measurements of its angles. To do this, we shall need to know the length of one side* along with (i) two angles; (ii) one angle and one other side; (iii) the other two sides. There are four possibilities to consider:

- CASE 1: One side and two angles are known (ASA or SAA).
 CASE 2: Two sides and the angle opposite one of them are known (SSA).
 CASE 3: Two sides and the included angle are known (SAS).
 CASE 4: Three are known (SSS).

The four cases are illustrated below.



Use the Law of Sines to solve triangles for
Case 1 or Case 2.

Use the Law of Cosines to solve
triangles for Case 3 or Case 4.

Law of Sines

For a triangle with sides a, b, c and opposite angles α, β, γ , respectively,

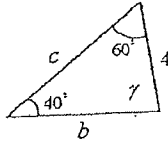
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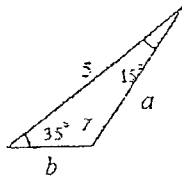
$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Example 1: Solve the triangle: $\alpha = 40^\circ$, $\beta = 60^\circ$, $a = 4$



Solution: $\gamma = 80^\circ$, $b = \frac{4 \sin 60^\circ}{\sin 40^\circ} \approx 5.39$, $c = \frac{4 \sin 80^\circ}{\sin 40^\circ} \approx 6.13$

Example 2: Solve the triangle: $\alpha = 35^\circ$, $\beta = 15^\circ$, $c = 5$



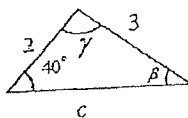
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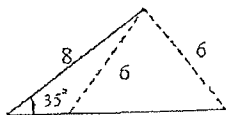
Solution: $\gamma = 130^\circ$, $a = \frac{5 \sin 35^\circ}{\sin 130^\circ} \approx 3.74$, $b = \frac{5 \sin 15^\circ}{\sin 130^\circ} \approx 1.69$

Example 3: Solve the triangle: $a = 3, b = 2, \alpha = 40^\circ$

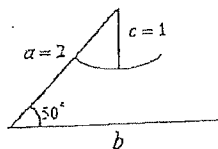


Solution: $\sin \beta \approx 0.43$, $\gamma = 154.6^\circ$, $c = \frac{3 \sin 114.6^\circ}{\sin 40^\circ} \approx 4.24$

Example 4: Solve the triangle: $a = 6, b = 8, \alpha = 35^\circ$



Solution: $\beta_1 \approx 49.9^\circ$ or $\beta_2 \approx 130.1^\circ$, $\gamma_1 \approx 95.1^\circ$ or $\gamma_2 \approx 14.9^\circ$, $c_1 \approx 10.42$ or $c_2 \approx 2.69$



Example 5: Solve the triangle: $a=2, c=1, \gamma=50^\circ$

Solution: $\sin \alpha = 2 \sin 50^\circ \approx 1.53$

1.18 Law of Sines Homework

1. The difference formula for the sine function is $\sin(\alpha - \beta) = \underline{\hspace{2cm}}$.
2. If θ is an acute angle, solve the equation $\cos \theta = \frac{\sqrt{3}}{2}$.
3. A triangle with sides 2 and 5 is similar to a triangle with corresponding sides of 3 and x . Find the missing lengths.
4. If none of the angles of a triangle is a right angle, the triangle is called .
5. For a triangle with sides a, b, c and opposite angles α, β, γ , the Law of Sines states that .
6. *True or False:* An oblique triangle in which two sides and an angle are given always results in at least one triangle.
7. *True or False:* The sum of the angles of any triangle equals 180° .

In problems 17 – 23, solve each triangle.

- | | |
|--|--|
| 17. $\alpha = 40^\circ, \beta = 20^\circ, a = 2$ | 19. $\beta = 70^\circ, \gamma = 10^\circ, b = 5$ |
| 21. $\alpha = 110^\circ, \gamma = 30^\circ, c = 3$ | 23. $\alpha = 40^\circ, \beta = 40^\circ, c = 2$ |

In problems 25 – 35, two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

25. $a=3, b=2, \alpha=50^\circ$ 27. $b=5, c=3, \beta=100^\circ$ 29. $a=4, c=6, \beta=20^\circ$
 31. $b=4, c=6, \beta=20^\circ$ 33. $a=2, c=1, \gamma=100^\circ$ 35. $a=2, c=1, \gamma=25^\circ$

1.19 Law of Cosines

We used the Law of Sines to solve Case 1 (SAA or ASA) and Case 2 (SSA) of an oblique triangle. Now we use the Law of Cosines to solve Case 3 and Case 4.

Case 3: Two sides and the including angle are known (SAS).

Case 4: Three sides are known (SSS).

Law of Cosines Theorem

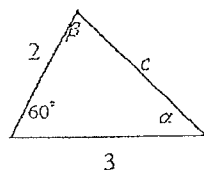
For a triangle with sides a, b, c and opposite angles α, β, γ respectively,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad (1)$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta \quad (2)$$

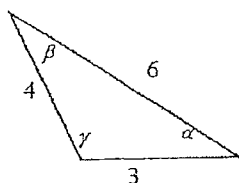
$$a^2 = b^2 + c^2 - 2bc \cos \alpha \quad (3)$$

Example 1 Solve a SAS Triangle where $a=2, b=3, \gamma=60^\circ$.



Solution: $c = \sqrt{7}$, $\alpha \approx 40.9^\circ$, $\beta \approx 79.1^\circ$

Example 2 Solve a SSS Triangle where $a = 4$, $b = 3$, $c = 6$.



Solution: $\alpha \approx 36.3^\circ$, $\beta \approx 26.4^\circ$, $\gamma \approx 117.3^\circ$

1.19 Law of Cosines Homework

1. Write the formula for the distance d from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$.
2. If θ is an acute angle, solve the equation $\cos \theta = \frac{\sqrt{2}}{2}$.
3. If three sides of a triangle are given, the Law of _____ is used to solve the triangle.
4. If one side and two angles of a triangle are given, the Law of _____ is used to solve the triangle.
5. If two sides and the included angle of a triangle are given, the Law of _____ is used to solve the triangle.
6. *True or False:* Given only the three sides of a triangle, there is insufficient information to solve the triangle.
7. *True or False:* Given two sides and the included angle, the first thing to do to solve the triangle is to use the Law of Sines.
8. *True or False:* A special case of the Law of Cosines is the Pythagorean Theorem.

In problems 17 – 31, solve each triangle.

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17. $a=3$, $b=4$, $\gamma=40^\circ$

19. $b=1$, $c=3$, $\alpha=80^\circ$

21. $a=3$, $c=2$, $\alpha=80^\circ$

23. $a=2$, $b=2$, $\gamma=50^\circ$

25. $a=12$, $b=13$, $c=5$

27. $a=2$, $b=2$, $c=2$

29. $a=5$, $b=8$, $c=9$

31. $a=10$, $b=8$, $c=5$

EXHIBIT 10

2.1.8 Trigonometric Functions of General Angles

Draw a negative angle where $(3, -4)$ is a point on the angle. Evaluate the six trigonometric functions. Hint: Start by drawing the angle and the corresponding triangle. Label the sides.

Evaluate the six trigonometric functions for each *quadrantal* angle (i.e. the angle on an axis).

(a) $\theta = 0 = 0^\circ$ (b) $\theta = \frac{\pi}{2} = 90^\circ$ (c) $\theta = \pi = 180^\circ$ (d) $\theta = \frac{3\pi}{2} = 270^\circ$

Answers:

a) $\sin 0 = \sin 0 = 0$

$\cos 0 = 1$

$\tan 0 = 0$

$\sec 0 = 1$

$\csc 0$ & $\cot 0$

are undefined

b) $\sin \frac{\pi}{2} = 1$

$\cos \frac{\pi}{2} = 0$

$\csc \frac{\pi}{2} = 1$

$\cot \frac{\pi}{2} = 0$

$\tan \frac{\pi}{2}$ and $\sec \frac{\pi}{2}$

are undefined

c) $\sin \pi = 0$

$\cos \pi = -1$

$\tan \pi = 0$

$\sec \pi = -1$

$\csc \pi$ & $\cot \pi$

are undefined

d) $\sin \frac{3\pi}{2} = -1$

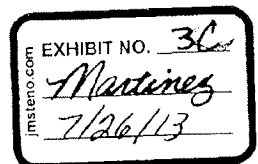
$\cos \frac{3\pi}{2} = 0$

$\csc \frac{3\pi}{2} = -1$

$\cot \frac{3\pi}{2} = 0$

$\tan \frac{3\pi}{2}$ & $\sec \frac{3\pi}{2}$

are undefined



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Evaluate the trigonometric function. Hint: Draw and label the triangle first.

a) $\sin(-330^\circ)$

b) $\cos 780^\circ$

The reference angle is 30° . $\therefore \sin(-330^\circ) = \sin 30^\circ = \frac{1}{2}$

c) $\tan \frac{17\pi}{4}$

d) $\sec\left(-\frac{15\pi}{4}\right)$

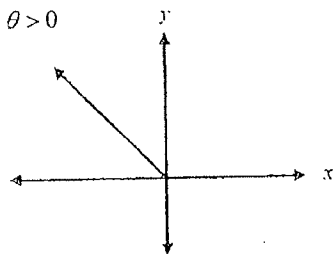
e) $\csc(450^\circ)$

Draw θ . In which quadrant does the angle θ lie? Identify the corresponding reference angle α .

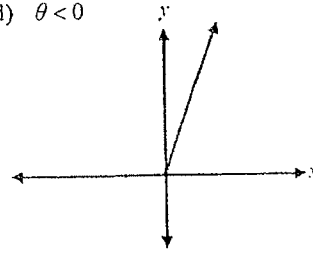
(a) $\tan \theta > 0$ and $\sec \theta < 0$

(b) $\sin \theta > 0$ and $\cos \theta < 0$

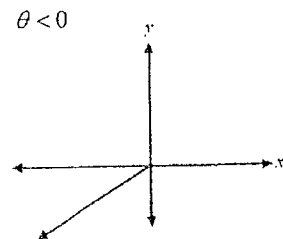
(c) $\theta > 0$



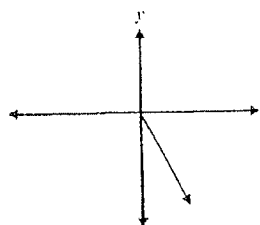
(d) $\theta < 0$



(e) $\theta < 0$



(f) $\theta > 0$



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Find the sine of the angle. First draw the angle; identify the reference angle; then label the corresponding right triangle.

(a) 120° (b) -135°

Identify the reference angle

Label the corresponding right triangle

State the sine

(c) $\frac{13\pi}{6}$ (d) $\frac{7\pi}{-3}$

Draw the reference angle

Label the corresponding right triangle

State the sine

Steps for finding the values of the trigonometric functions of an angle

If the angle θ is on the x -axis or the y -axis, draw the x -axis and y -axis, then

1st label the coordinates of a point one unit from the origin 2nd name the trig value.

If the angle θ is in a quadrant, draw the x -axis and y -axis, then

1st draw the angle 2nd label the corresponding right triangle 3rd name the trig value.

Find the value of the following trigonometric functions.

(a) $\sin -135^\circ$ (b) $\csc 780^\circ$ (c) $\sec \frac{7\pi}{-6}$ (d) $\tan \left(\frac{15\pi}{4} \right)$

Given the $\tan \theta = 1$ and the $\sin \theta < 0$, find the value of the remaining trig functions.

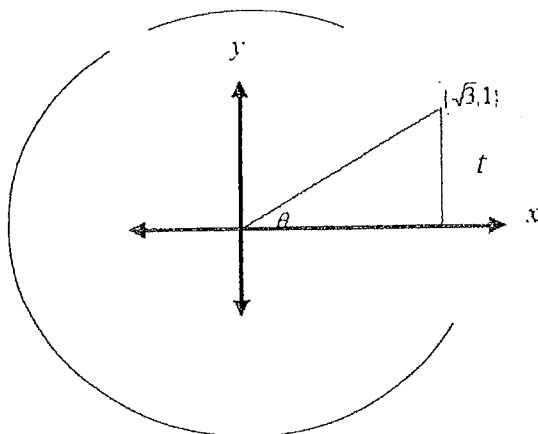
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2.2.9 Circle Trigonometry

Let t be the length of the arc from the x -axis to the point $P = (\sqrt{3}, 1)$ on a circle.

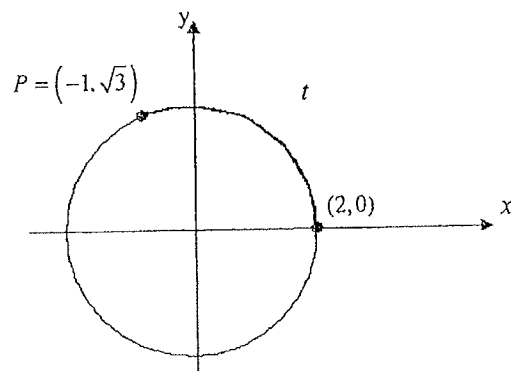


$$\theta = t \text{ radians}$$

$$\sin \theta = \sin t = \frac{1}{2}$$

$$\cos \theta = \cos t = \frac{\sqrt{3}}{2}$$

Identify the reference angle, label the triangle, then evaluate the 6 trigonometric functions.



The unit circle has a radius of 1.
 This is not the unit circle.
 When might we use a unit circle?
 When working with quadrantal angles.

Domain & Range of Trigonometric Functions

Function	The Domain is the angle	The Range is the ratio.
$f(x) = \sin x$	$x = \text{All real numbers}$	The Range is the function value. $\sin x = [-1 \text{ to } 1]$
$f(x) = \cos x$	$x = \text{All real numbers}$	$\cos x = [-1 \text{ to } 1]$
$f(x) = \tan x$	$x = (-\infty, +\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\tan x = (-\infty, +\infty)$
$f(x) = \csc x$	$x = (-\infty, +\infty)$, except integer multiples of π	$\csc x \leq -1, \csc x \geq 1$
$f(x) = \sec x$	$x = (-\infty, +\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\sec x \leq -1, \sec x \geq 1$
$f(x) = \cot x$	$x = (-\infty, +\infty)$, except integer multiples of π	$\cot x = (-\infty, +\infty)$

Periodic functions repeat after a certain time (period).

All the trigonometric functions repeat, so all trig functions are periodic functions.

2π is the repeating period for sine, cosine, cosecant, and secant.

$$\begin{aligned} \sin(\theta + 2\pi) &= \sin \theta & \cos(\theta + 2\pi) &= \cos \theta \\ \csc(\theta + 2\pi) &= \csc \theta & \sec(\theta + 2\pi) &= \sec \theta \end{aligned}$$

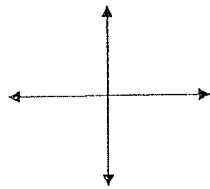
π is the period for tangent and cotangent.

$$\begin{aligned} \tan(\theta + \pi) &= \tan \theta \\ \cot(\theta + \pi) &= \cot \theta \end{aligned}$$

Find the exact value of:

a) $\cos 390^\circ$

b) $\cot -\frac{3\pi}{4}$



c) $\sin \frac{15\pi}{4}$

d) $\csc(-660)$

Even and Odd Functions

1) Even functions are symmetric about the y-axis.

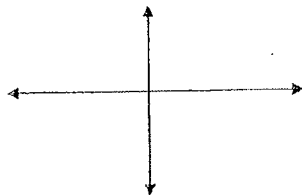
2) A function is even if $f(-x) = f(x)$
 $y = \cos(-x)$ is the same as $y = \cos(x)$

i.e. the graph equations are equal

3) $(-x, y)$ and (x, y) are on the graph.

The sign of the x changes but the y doesn't and the two points are on the graph.

$$\cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) =$$



The points are $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

1) Odd functions are symmetric about the origin.

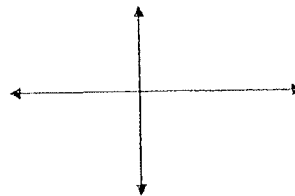
2) A function is odd if $-f(-x) = f(x)$
 $-y = \sin(-x)$ is the same as $y = \sin(x)$

i.e. the graph equations are equal

3) $(-x, -y)$ and (x, y) are on the graph.

The sign of the x and y changes, and the two points are on the graph.

$$\sin\left(-\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) =$$



The cosine and the secant are even functions:	$\cos(-\theta) = \cos \theta$	$\sec(-\theta) = \sec \theta$
The rest are odd functions:	$\sin(-\theta) = -\sin \theta$	$\tan(-\theta) = -\tan \theta$
	$\csc(-\theta) = -\csc \theta$	$\cot(-\theta) = -\cot \theta$

Find the exact value of the functions. Show/explain why the function is even or odd.

- 1) $\sec(-135^\circ)$ 2) $\sin(-45^\circ)$ 3) $\csc\left(\frac{3\pi}{4}\right)$ 4) $\cos(-\pi)$

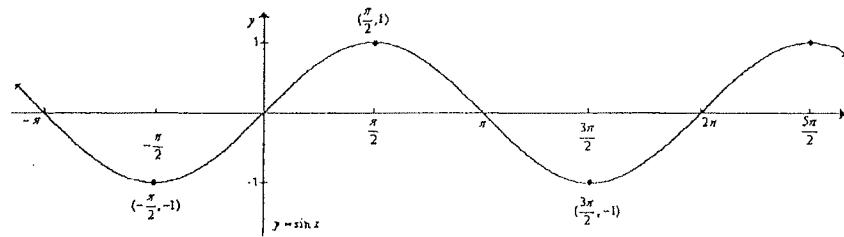
- 5) $\sin(-3\pi)$ 6) $\cos\left(\frac{\pi}{6}\right)$ 7) $\cot\left(-\frac{3\pi}{2}\right)$ 8) $\tan\left(\frac{5\pi}{2}\right)$

- 9) $\tan\left(-\frac{37\pi}{4}\right)$ 10) $\cot(-390^\circ)$ 11) $\csc 420^\circ$ 12) $\sec 270^\circ$

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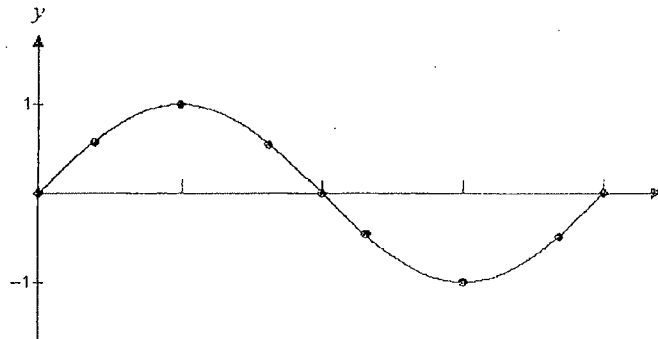
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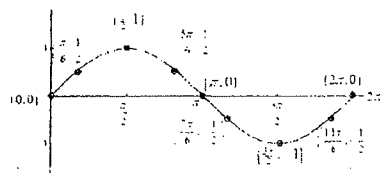
2.3.10 Graphs & Transformations of the Sine and Cosine**Graph of the Sine Function** $y = \sin x$, $-\infty \leq x \leq +\infty$ 

angle "x"	y sin x
-----------	------------

0

 $y = \sin x$, $0 \leq x \leq 2\pi$ The period of the sine and the cosine is 2π .

y = sin x, 0 ≤ x ≤ 2π



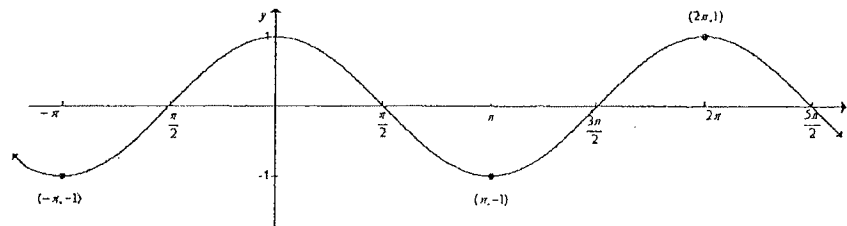
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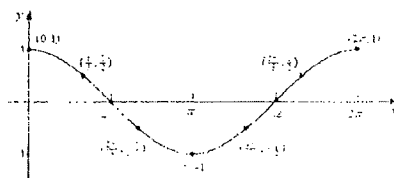
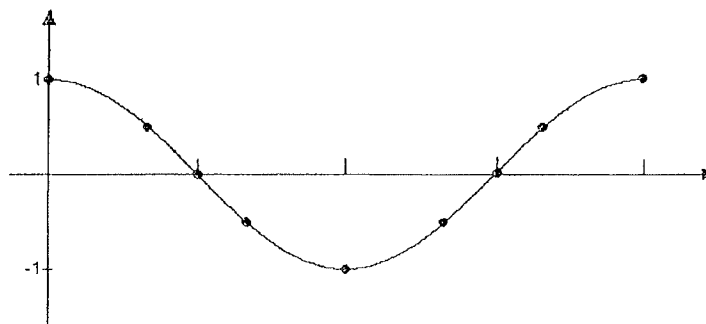
Graphs of the Cosine Function

$y = \cos x, -\infty \leq x \leq +\infty$



angle "x"	y cos x
0	

$y = \cos x, 0 \leq x \leq 2\pi$



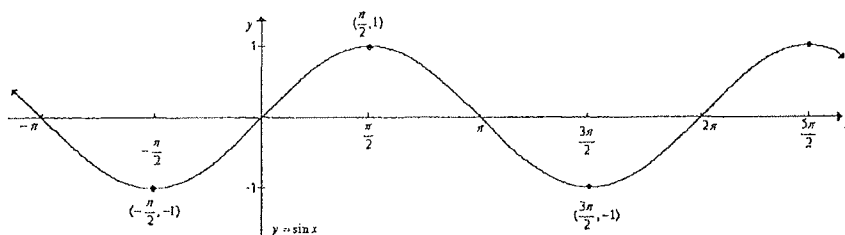
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Horizontal shifting of the graph.Use the graph of $y = \sin x$ to graph $y = \sin\left(x - \frac{\pi}{4}\right)$.

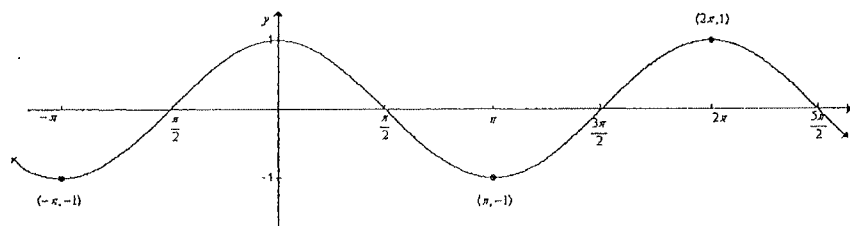
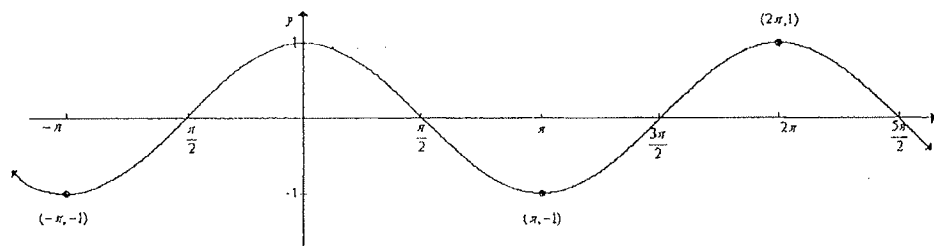
Hint: plot the maximums, minimums, zeros.

**The Amplitude is the coefficient of the trig function.**

$$y = A \cos(\omega x - \varphi)$$

Use the graph of $y = \cos x$ to graph $y = 2 \cos x$

Hint: plot the maximums, minimums, zeros.

Use the Graph of $y = \cos x$ to graph $y = \frac{1}{2} \cos x$.

"1" is the amplitude (i.e. the largest vertical value) of the sine and cosine functions.

"2π" is the period of the sine, cosine, csc, sec functions.

"π" is the period of the tangent and the cotangent functions.

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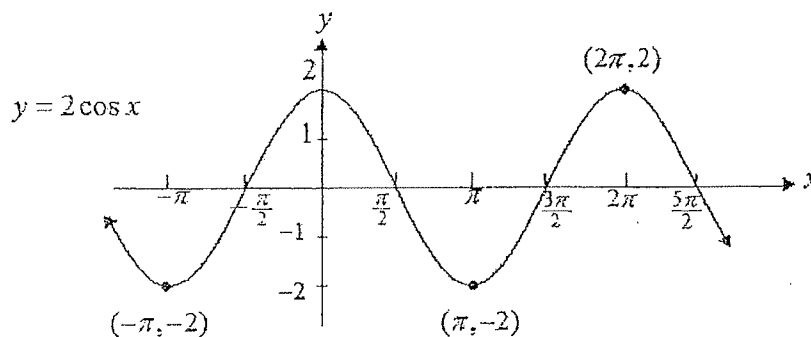
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The Domain, Range, Amplitude, and Period of a graph $y = A \cos(\omega x)$ $y = \frac{1}{3} \cos(2x)$ Earlier we graphed $y = 2 \cos x$, which is reproduced below.

The domain is the horizontal spread of the graph from left to right.

 $\text{Domain} = \{x | x = (-\infty, +\infty)\}$ The domain is the set of all x -values such that x is a value between $-\infty$ and $+\infty$ Notice that the y -values of $y = 2 \cos x$ lie between -2 and 2 , including the endpoints. $\text{Range} = \{y | y = [-2, +2]\}$ The range is the set of all y -values such that y is a value between -2 and 2 , including endpoints.The amplitude is the graph's largest vertical distance from the x -axis. Distance is always positive. $\text{Amplitude} = 2$

The period is the horizontal length of the interval that is repeated in the graph.

 $\text{Period} = 2\pi$ The most general sine and cosine function (when ω is positive) is
 $y = A \sin \omega(x - \phi) + D$ and $y = A \cos \omega(x - \phi) + D$

$$y = 2 \cos x$$

$$y = \frac{1}{2} \cos x$$

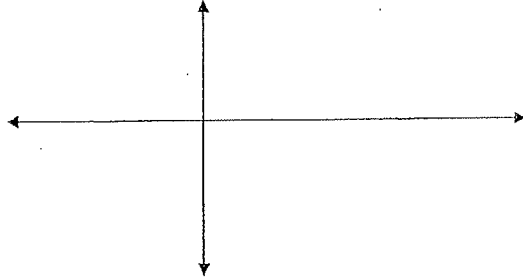
$$y = 3 \sin \left(x - \frac{\pi}{4} \right)$$

The Amplitude of the sine and cosine is $|A|$ ~ the range

Sine & Cosine Period = $\left| \frac{2\pi}{\omega} \right|$

Note: When ω is one, then $y = A \cos (x - \phi) + D$ $|A|$ is *amplitude* 2π is the *period* ϕ is the *horizontal shift* D is the *vertical shift*

Identify the amplitude and period of $y = 2 \sin(\pi x)$, then graph at least 2 periods.



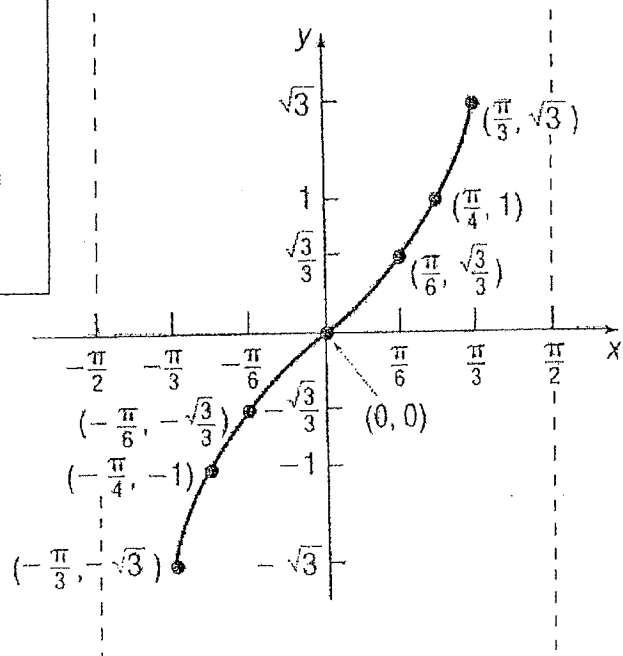
Identify the amplitude and period of $y = -4 \cos(x)$, and graph at least 2 periods.

2.4.11 Graphing the Tangent, Cotangent, Cosecant, Secant

Graph one period of the Tangent Function

$$y = \tan x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

the angle x	the ratio value $y = \tan x$	Plot this point (x, y)
$-\frac{\pi}{2}$		
$-\frac{\pi}{3}$	$-\sqrt{3} \approx -1.73$	$(-\frac{\pi}{3}, -1.73)$
$-\frac{\pi}{4}$	-1	$(-\frac{\pi}{4}, -1)$
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \approx -0.58$	$(-\frac{\pi}{6}, -0.58)$
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.58$	$(\frac{\pi}{6}, .58)$
$\frac{\pi}{4}$	1	$(\frac{\pi}{4}, 1)$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.73$	$(\frac{\pi}{3}, 1.73)$
$\frac{\pi}{2}$		



The Tangent Function has Asymptotes

@ its discontinuities:

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots, \frac{(2n-1)\pi}{2}$$

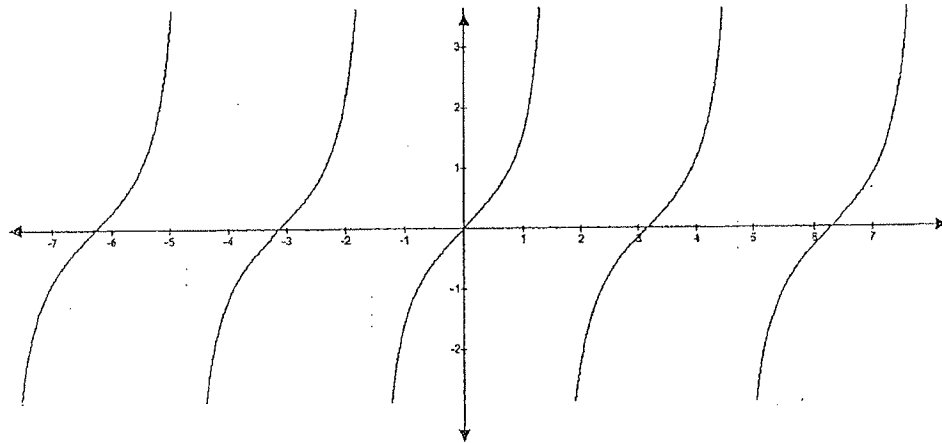
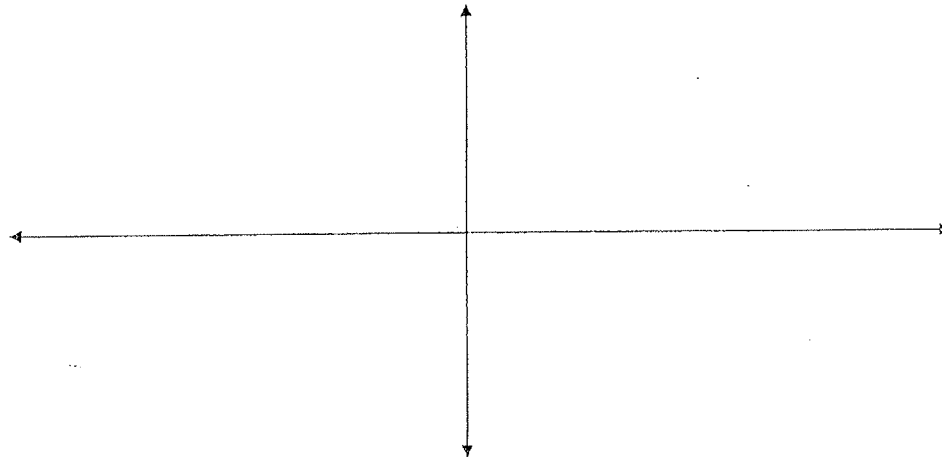
$$x = -\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, -\frac{7\pi}{2}, \dots, -\frac{(2n-1)\pi}{2}$$

natural # $n = 1, 2, 3, \dots$ $2n$ = an even number $2n-1$ = an odd number

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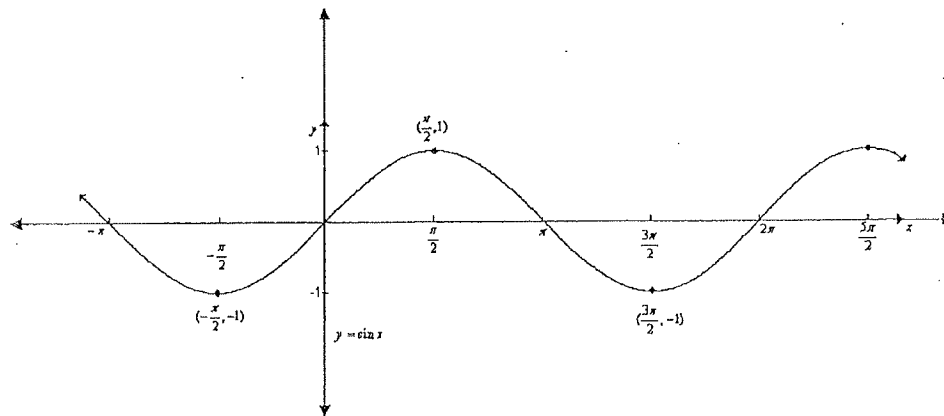
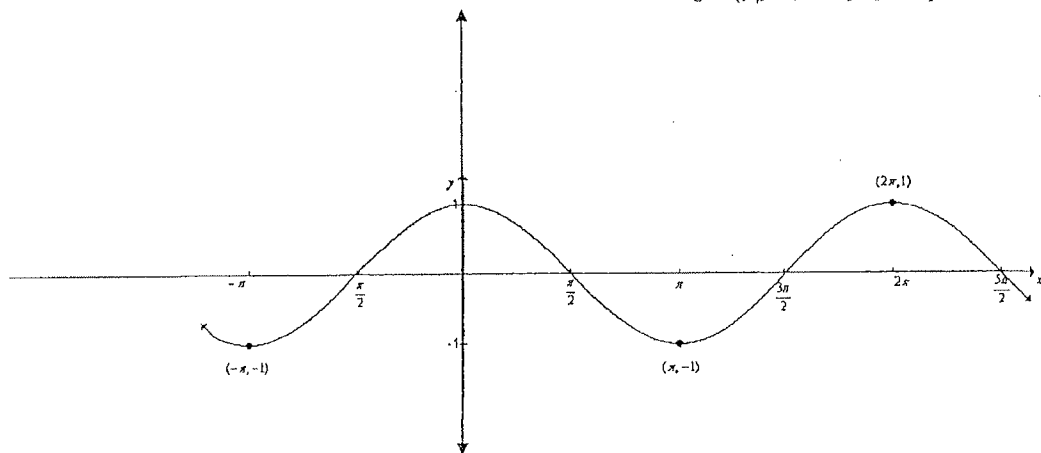
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Graph the Tangent $y = \tan x$, $x = (-\infty, +\infty)$ Show at least 4 periods.Asymptotes @ $x = \frac{\pi}{2}$, $x = \frac{\pi}{2} \pm n\pi$ Domain = $\{x \mid x = \text{all real numbers except } x \neq \frac{\pi}{2}, x \neq \frac{\pi}{2} \pm n\pi\}$ Range = $\{y \mid y = (-\infty, +\infty)\}$ **Graph the Cotangent** $y = \cot x$ Show at least 4 periods. Asymptotes at $x = n\pi$ where $n = \text{all whole numbers}$ Asymptotes @ $x = 0$, $x = \pm n\pi$ Domain = $\{x \mid x = \text{all real numbers except } x \neq 0, x \neq \pm n\pi\}$ Range = $\{y \mid y = (-\infty, +\infty)\}$ 

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Graph the Cosecant $y = \csc x$ There are asymptotes at the zeros of the sine function, same as the cotangent asymptotes.Asymptotes @ $x = 0, x = \pm n\pi$ Domain = $\{x \mid x = \text{all real numbers except } x \neq 0, x \neq \pm n\pi\}$ Range = $\{y \mid y = (-\infty, -1] \cup [1, +\infty)\}$ **Graph the Secant** $y = \sec x$ There are asymptotes at the zeros of the cosine function, same as the tangent asymptotes.Asymptotes @ $x = \frac{\pi}{2}, x = \frac{\pi}{2} \pm n\pi$ Domain = $\{x \mid x = \text{all real numbers except } x \neq \frac{\pi}{2}, x \neq \frac{\pi}{2} \pm n\pi\}$ Range = $\{y \mid y = (-\infty, -1] \cup [1, +\infty)\}$ 

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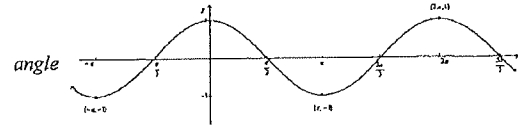
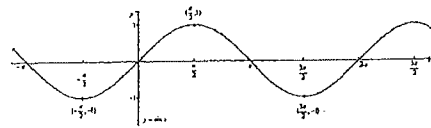
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Summary of Trigonometric Graphs*y is the ratio of the trig function*

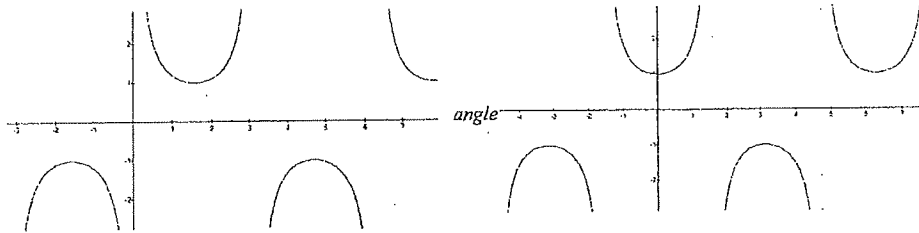
$$y = \sin x, \quad -\infty \leq x \leq +\infty$$

$$y = \cos x, \quad -\infty \leq x \leq +\infty$$



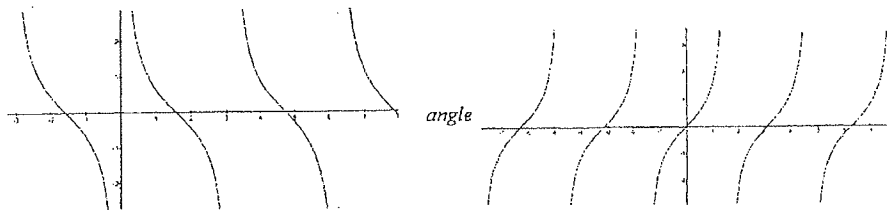
$$y = \csc x, \quad x = [-\infty, +\infty]$$

$$y = \sec x, \quad x = [-\infty, +\infty]$$



$$y = \cot x, \quad x = (-\infty, +\infty)$$

$$y = \tan x, \quad x = (-\infty, +\infty)$$



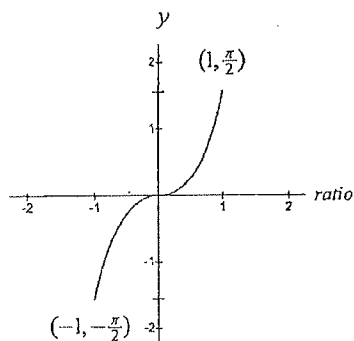
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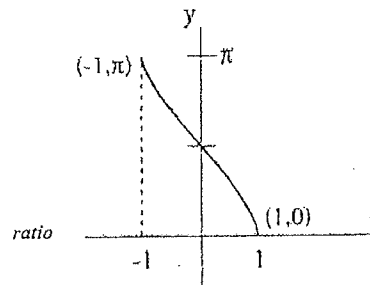
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2.5.12 Inverse Trigonometric Functions $y = \arcsin x$, $y = \arccos x$, $y = \arctan x$ Summary of Inverse Trigonometric Graphs y is the angle of the trig function

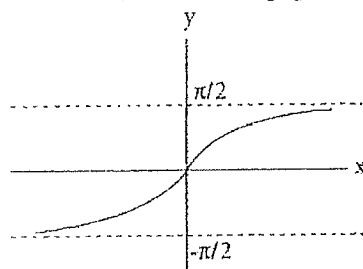
$$y = \arcsin x, \quad x = [-1, 1]$$

 $y = \text{the angle which is } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \text{ in Q1 or 4}$ 

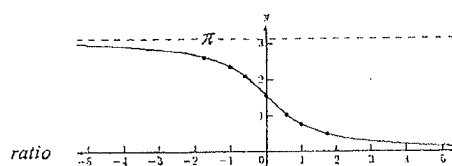
$$y = \arccos x, \quad x = [-1, 1]$$

 $y = \text{the angle which is } [0, \pi], \text{ in Q1 or 2}$ 

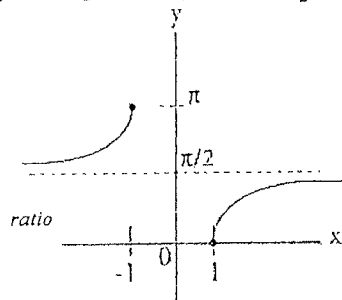
$$y = \arctan x, \quad x = \text{all real numbers}$$

 $y = \text{the angle which is } (-\frac{\pi}{2}, \frac{\pi}{2}), \text{ in Q1 or 4}$ 

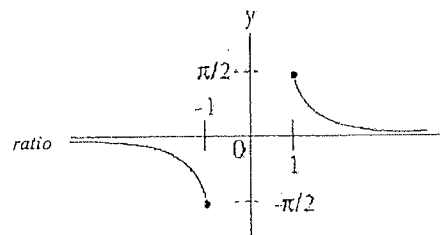
$$y = \text{arccot } x, \quad x = \text{all real numbers}$$

 $y = \text{the angle which is } (0, \pi), \text{ in Q1 or 2}$ 

$$y = \text{arcsec } x, \quad x \neq (-1, 1)$$

 $y = \text{the angle which is } [0, \pi] \text{ but } y \neq \frac{\pi}{2}, \text{ in Q1 or 2}$ 

$$y = \text{arccsc } x, \quad x \neq (-1, 1)$$

 $y = \text{the angle which is } [-\frac{\pi}{2}, \frac{\pi}{2}] \text{ but } y \neq 0, \text{ in Q1 or 4}$ 

We will first study the Inverse Sine, Cosine, and Tangent Functions

Trigonometry is comprised of doing two things: Determining the trig ratio or identifying the angle. This last one is called finding the inverse.

Normally, we need to determine **the trig ratio**.

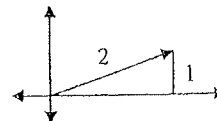
Example: Give the six trig ratios for the angle $\theta = \frac{\pi}{3}$.

Finding the angle is called doing **the inverse**. Why? Because you already know the trig ratio so you need to inverse your action.

Example: Find the angle when the sine ratio is $\frac{1}{2}$.

$\sin \theta = \frac{1}{2}$ θ is the angle whose sine ratio is $\frac{1}{2}$. In math symbols this looks like

$$\theta = \arcsin \frac{1}{2} \quad \text{or} \quad \theta = \sin^{-1} \left(\frac{1}{2} \right)$$



This is a $30^\circ - 60^\circ - 90^\circ$ triangle. \therefore the angle θ is 30° .

Without a calculator, evaluate each of the following inverse trig functions (i.e. find the angle).

$$\sin^{-1} \frac{\sqrt{3}}{2}$$

$$\cos^{-1} \left(-\frac{\sqrt{2}}{2} \right)$$

$$\arctan 1$$

$$\arcsin \left(-\frac{\sqrt{2}}{2} \right)$$

$$\arctan(0)$$

$$\arccos(-1)$$

Use your calculator to find the angle in radians and in degrees (to 3 decimal places):

$$\sin^{-1} \frac{2}{3}$$

$$\tan^{-1} 2.48$$

$$\sin^{-1} \left(-\frac{3}{4} \right)$$

$$\arctan \left(\frac{\sqrt{3}}{2} \right)$$

$$\arctan 100$$

Answer: .730;
41.810^o

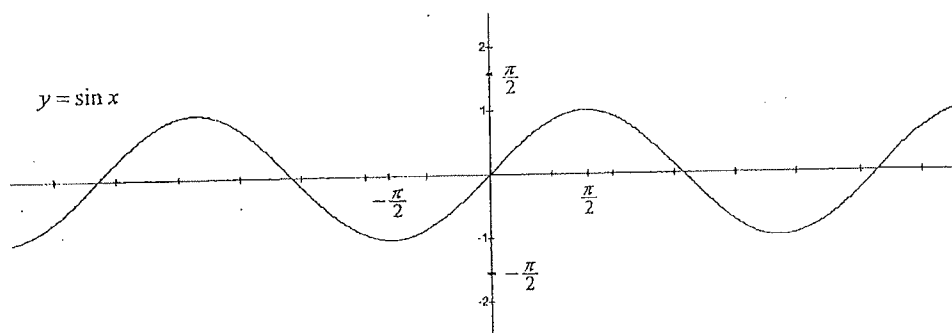
1.188;
68.039^o

1.561
89.427^o

Graph of the Inverse Sine Function

We use part of the sine function to get the inverse sine function.

i.e. We use the domain from $[-\frac{\pi}{2}$ to $\frac{\pi}{2}]$ which becomes the range of the inverse cosine function.



The inverse sine function: $y = \sin^{-1} x$ means that $x = \sin y$

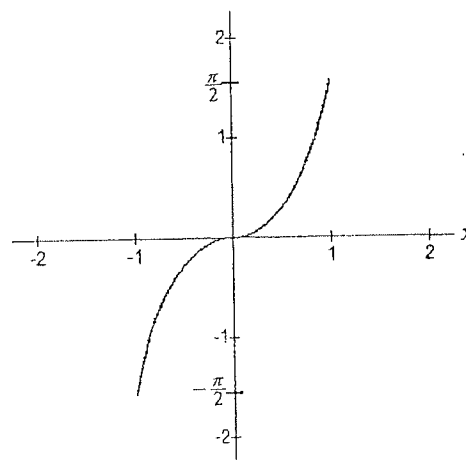
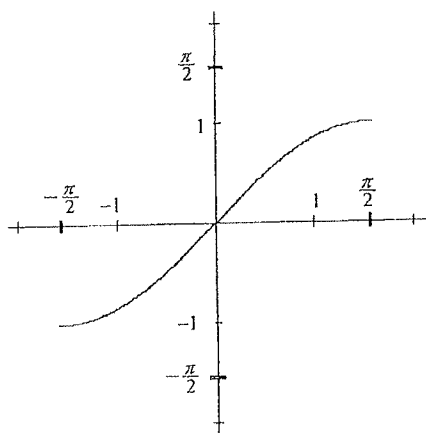
The Domain is $x = [-1, +1]$

The Range is *the angle* $y = [-\frac{\pi}{2}, +\frac{\pi}{2}]$

Angle is in Quadrant 1 or 4

$y = \arcsin x, \quad x = [-1, 1]$

$y = \sin x, \quad x = [-\frac{\pi}{2}, \frac{\pi}{2}]$



$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x$$

↙
angle = $[-\frac{\pi}{2}, +\frac{\pi}{2}]$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x$$

↙
ratio = $[-1, +1]$

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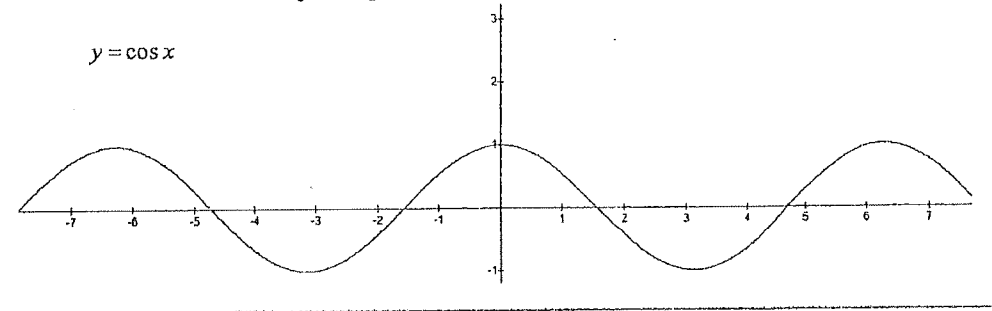
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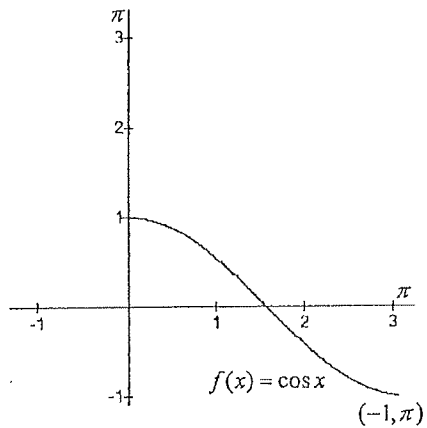
The Inverse Cosine Function

We use part of the cosine function to get the inverse cosine function.

i.e. We use the domain from $[0 \text{ to } \pi]$ which becomes the range of the inverse cosine function.



y is the cosine ratio



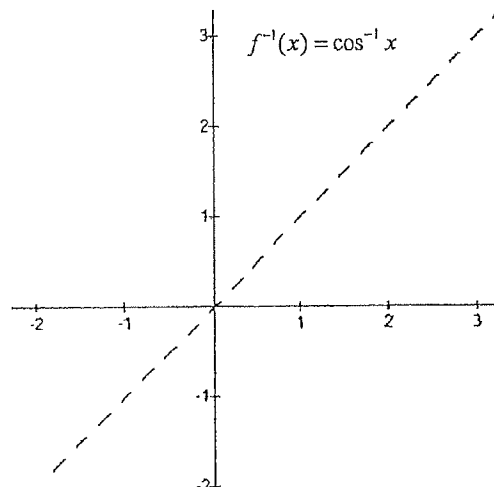
$$y = \cos x$$

$$f(x) = \cos x$$

Limited Domain: $x = [0, \pi]$ i.e. the angle

Range = $\cos x = [-1 \text{ to } 1]$

y is the angle = $[0, \pi]$

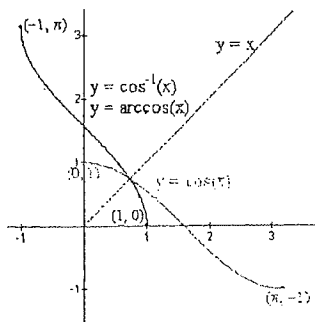


$$y = \cos^{-1} x = \text{the angle} = [0, \pi]$$

$$f^{-1}(x) = \cos^{-1} x$$

Domain = $x = [-1, +1]$

Range = $\text{angle} = [0, \pi]$ in Quadrant 1 or 2



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$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x$$

↙
angle = $[0, \pi]$

$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x$$

↙
ratio = $[-1, +1]$

Evaluate without using a calculator. Indicate if the answer is a ratio (number) or an angle.

a) $\cos(\arcsin 0.5) \approx 0.8660$ b) $\cos[\arctan(-1)]$ c) $\cos[\arctan(-1)] = \frac{\sqrt{2}}{2}$

d) $\cos^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$ e) $\cos\left[\arcsin\left(-\frac{3}{5}\right)\right]$ f) $\sin\left[\arcsin\left(\frac{2}{3}\right)\right]$

Solution d) angle, $\frac{\pi}{3}$

e) ratio, $\frac{4}{5}$

f) ratio, $\frac{2}{3}$

g) $\cos(\arctan 1)$

h) $\tan(\arccos \frac{2}{3})$

i) $\sin\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$

$\frac{\sqrt{5}}{2}$

Solve:

$$6 \sin x = \pi$$

Solve:

$$6 \cos x = \pi$$

$$\cos^{-1} \frac{\pi}{6}$$

$$\frac{1}{\cos \frac{\pi}{6}} = \left(\cos \frac{\pi}{6}\right)^{-1}$$

$$58^\circ$$

$$\frac{2}{\sqrt{3}} \approx 1.2$$

Solve: $6 \sin^{-1} x = \pi$

Solve: $6 \cos^{-1} x = \pi$

Solve: $2 \cos t = \sqrt{3}$

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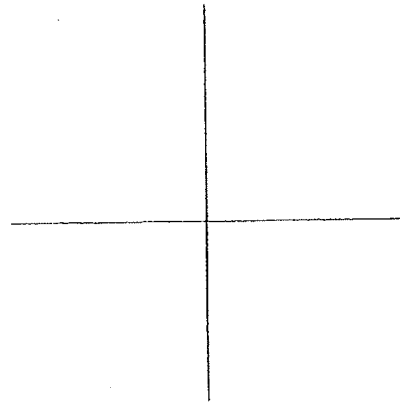
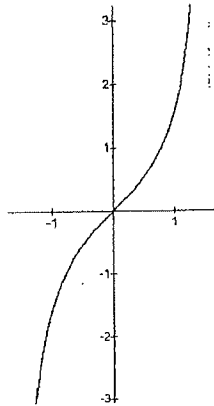
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The ratio is $\frac{1}{2}$ The ratio is $\frac{\sqrt{3}}{2}$ **The Inverse Tangent Function**

$$y = \tan^{-1} x = \arctan x \text{ means } x = \tan y$$

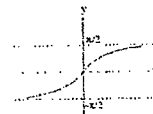
The angle is $y = (-\frac{\pi}{2}, +\frac{\pi}{2})$, x is the tangent ratio $\in (-\infty, +\infty)$

The restricted tangent function



$$\begin{aligned} & \tan(\arctan x) \\ f(f^{-1}(x)) &= \tan(\tan^{-1} x) = x \\ & \text{ratio} = (-\infty, +\infty) \end{aligned}$$

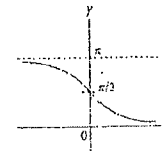
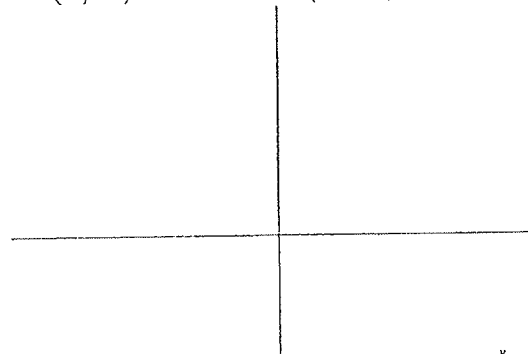
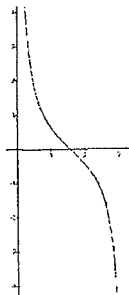
$$\begin{aligned} & \arctan(\tan x) \\ f^{-1}(f(x)) &= \tan^{-1}(\tan x) = x \\ & \text{angle} = [-\frac{\pi}{2}, +\frac{\pi}{2}] \end{aligned}$$

**The Inverse Cotangent Function**

$$y = \cot^{-1} x = \operatorname{arccot} x \text{ which means } x = \cot y$$

 y is the angle and x is the cotangent ratio. $(0, \pi)$ between $(-\infty, +\infty)$.

The restricted cotangent function



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Evaluate the expressions.

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$$\cot^{-1}\left(\frac{\pi}{6}\right)$$

$$\therefore \frac{\pi}{6} = \cot y$$

Q1or2

$$\tan y = \frac{6}{\pi}$$

$$y = \arctan \frac{6}{\pi}$$

The angle ≈ 1.088 or 62.4°

$$\frac{1}{\cot\left(\frac{\pi}{6}\right)}$$

$$\cot\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

the reciprocal is

$$\frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$$

$$y = \arccot \frac{\sqrt{3}}{3} \quad \cot^{-1}\left(\cos \frac{2\pi}{3}\right)$$

$$\frac{\pi}{3}$$

The angle $\approx .4636 \approx 26.6^\circ$

Inverse Cosecant Function $y = \csc^{-1} x$, $x \leq -1$ or $x \geq +1$, and $y = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$, $y \neq 0$

$$x = \csc y$$

$$y = \csc^{-1}(-2)$$

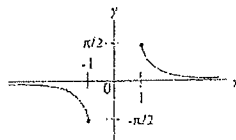
$$\operatorname{arccsc}(-\sqrt{2})$$

$$\csc^{-1}(-3)$$

$$\csc^{-1}(1.3465)$$

$$\therefore -2 = \csc y$$

Q1or4



$$\sin y = \frac{-1}{2}$$

$$\frac{\pi}{6}$$

$$-\frac{\pi}{4}$$

$$\approx 19.5^\circ \approx -.34$$

$$\approx 48.0^\circ \approx .8310$$

The Inverse Secant Function $y = \sec^{-1} x$ $x \leq -1$ or $x \geq +1$, and $\theta = y = [0, \pi]$, $y \neq \frac{\pi}{2}$

$$x = \sec y$$

$$\operatorname{arcsec} 2$$

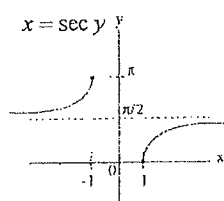
$$\sec^{-1} 2$$

$$\operatorname{arccot}(-\sqrt{3})$$

$$\cot(\csc^{-1} 2)$$

$$\therefore 2 = \sec y$$

Q1or2



$$\cos y = \frac{1}{2}$$

$$\frac{\pi}{3}$$

$$\frac{\pi}{3}$$

$$\sqrt{3}$$

$$\sqrt{3}$$

Summary:

The Range is the angle y

The Domain is the ratio.

$$\text{The angle is } y = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$$

$$y = \sin^{-1} x$$

$$x = [-1, +1]$$

$$\text{The angle is } y = [0, \pi]$$

$$y = \cos^{-1} x$$

$$x = [-1, +1]$$

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The angle is $y = \left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$

$$y = \tan^{-1} x$$

$$x = (-\infty, +\infty)$$

The angle is $y = [0, \pi]$, $y \neq \frac{\pi}{2}$

$$y = \sec^{-1} x,$$

$$x \leq -1 \text{ or } x \geq 1$$

The angle is $y = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$, $y \neq 0$

$$y = \csc^{-1} x$$

$$x \leq -1 \text{ or } x \geq +1$$

The angle is $y = (0, \pi)$

$$y = \cot^{-1} x$$

$$x = (-\infty, +\infty)$$

Summary of Composite Trig Functions

$$f(f^{-1}(x)) = \sin(\arcsin x) = x$$

$$\text{ratio} = [-1, +1]$$

$$f^{-1}(f(x)) = \arcsin(\sin x) = x$$

$$\text{angle} = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$$

$$f(f^{-1}(x)) = \cos(\arccos x) = x$$

$$\text{ratio} = [-1, +1]$$

$$f^{-1}(f(x)) = \arccos(\cos x) = x$$

$$\text{angle} = [0, \pi]$$

$$f(f^{-1}(x)) = \tan(\arctan x) = x$$

$$\text{ratio} = (-\infty, +\infty)$$

$$f^{-1}(f(x)) = \arctan(\tan x) = x$$

$$\text{angle} = \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$$

2.6.13 Values of Inverse Trig Functions

For finding $\operatorname{arccot} x$ with a calculator, rewrite the function in terms of $\arctan x$

For finding $\operatorname{arcsec} x$, rewrite the function in terms of $\arccos x$

For finding $\operatorname{arccsc} x$, rewrite the function in terms of $\arcsin x$

$y = \operatorname{arcsec} x \Rightarrow \sec y = x$ which needs to be written as a cosine function.

$$\sec y = \frac{1}{\cos y} = x \Rightarrow \cos y = \frac{1}{x} \Rightarrow y = \arccos \frac{1}{x}$$

Evaluate the following expressions.

$$\cos(\arccos .5)$$

$$\tan(\arctan 2)$$

$$\cos(\arccos \pi)$$

$\arccos \pi$ is not defined

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 $\therefore \cos(\arccos \pi)$ is not defined

$$\cos(\arcsin \frac{7}{25})$$

$$\arcsin\left(\sin \frac{5\pi}{3}\right)$$

$$\frac{24}{25}$$

$$= \arcsin\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$$

$$\arcsin(\sin \frac{3\pi}{2})$$

$$\tan(\arctan(-5))$$

$$\cos(\arcsin \frac{1}{3})$$

$$-\frac{\pi}{2} \text{ not } \frac{3\pi}{2}$$

$$-5$$

$$\frac{2\sqrt{2}}{3}$$

Use a calculator to evaluate in radians to three decimal places:

a) $\operatorname{arccot}\left(\frac{1}{2}\right)$

b) $\cot^{-1}(-2)$

Evaluate without a calculator.

$$\cos(\arctan \sqrt{3})$$

$$\sin\left(\tan^{-1} \frac{3}{2}\right)$$

$$\frac{1}{2}$$

$$\frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

There comes a time
when one must take a
position that is
neither safe,

nor polític, nor
popular, but he must
take it because his
conscience tells him
it is right....

Martin Luther

3.1.14 Trigonometric Identities

Techniques used to Simplify Trigonometric Expressions.

- a) Simplify by rewriting in terms of sine and cosine functions.

$$\sec x \cos x$$

$$\sin x + \cot x \cos x$$

$$\csc \theta$$

- b) Simplify by first using a trig identity

$$\frac{\sec^2 \theta - 1}{\sin^2 \theta}$$

- c) Simplify by getting a common denominator

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta$$

Simplify by factoring.

$$c) \frac{\cot^2 \theta - \cot \theta - 2}{\cot \theta - 2}$$

$$\cot \theta + 1$$

d) Write without a fraction by multiplying the numerator & denominator by conjugate

$$\frac{1}{1 + \sin \theta}$$

$$\sec^2 \theta - \tan \theta \sec \theta$$

Techniques used to Simplify Trigonometric Equations:

Choose one side and make that side look like the other side.

1. Start with the more difficult side of the equation.
2. Combine fractions by adding or subtracting fractions (as indicated).
3. Change one side to sines and cosines.
4. Always remember what the other side looks like so you know where you are going.

Verify that the two sides are equal.

$$\sin x \cot x \sec x = 1$$

Hint: Change to sines and cosines

$$\frac{\sin x + \tan x}{1 + \cos x} = \tan x$$

Verify the identity: $\csc^2 x - \cos^2 x \csc^2 x = 1$

Hint: Factor

Verify the identity: $1 - 2\sin^2 x = 2\cos^2 x - 1$ Hint: Use a Pythagorean Identity.

Verify the identity. $\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$ Hint: Multiply by a conjugate.

3.2.15 The Sum and Difference Formulas

Formulas for the Cosine of a Sum and the Cosine of a Difference

The cosine of the sum of two angles equals the cosine of the first angle times the cosine of the second angle minus the sine of the first angle times the sine of the second angle.

Memorize these formulas
$$\begin{cases} \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{cases}$$

The cosine of the difference of two angles equals the cosine of the first angle times the cosine of the second angle plus the sine of the first angle times the sine of the second angle.

Verify the equation: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{6}$

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Verify the equation: $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{6}$

Verify the equation: $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{3}$

Evaluate $\cos 15^\circ$.

Hint: $15^\circ = 60^\circ - 45^\circ$

Solution: $\frac{\sqrt{2} + \sqrt{6}}{4}$

Find the exact value of $\cos \frac{\pi}{12}$.

Hint: $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

Solution: $\frac{1}{4}(\sqrt{6} + \sqrt{2})$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$

Prove the cofunction identity: $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ *Cofunctions of complementary angles are equal.*

Technique used: Make the left side (which is more complicated) look like the right side.

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \quad \text{Apply the } \cos(\alpha - \beta) \text{ formula.}$$

$$\sin x = \cos \frac{\pi}{2} \cos x + \sin \frac{\pi}{2} \sin x$$

$$\sin x = 0 \cdot \cos x + 1 \cdot \sin x$$

$$\sin x = \sin x \quad \text{Q.E.D.}$$

Prove the cofunction identity: $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ *Cofunctions of complementary angles are equal.*

Technique used: Make the left side (which is more complicated) look like the right side.

$$\cos x = \sin\left(\frac{\pi}{2} - x\right) \quad \text{Apply the cofunction of complementary angles theorem.}$$

$$\cos x = \cos\left[\frac{\pi}{2} - \left(\frac{\pi}{2} - x\right)\right]$$

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$$\cos x = \cos \left[\frac{\pi}{2} - \frac{\pi}{2} + x \right]$$

$$\cos x = \cos x$$

Q.E.D.

Verify the identity $\cos(\pi - \theta) = -\cos \theta$

Verify the identity

$$\frac{\cos 4x}{\sin x} - \frac{\sin 4x}{\cos x} = \frac{\cos 5x}{\sin x \cos x}$$

Hint: Subtract the fractions on the left. You'll see an identity.

Sum and Difference Formulas for the Sine Function

Memorize these formulas	$\begin{cases} \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \end{cases}$	The sine of a sum formula.
		The sine of a difference formula.

$$\text{Proof of } \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

Using the cofunction formula for cosine, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$, in reverse and $\theta = (\alpha + \beta)$ gives us

$$\begin{aligned} \sin(\alpha + \beta) &= \cos\left(\frac{\pi}{2} - (\alpha + \beta)\right) = \cos\left(\left(\frac{\pi}{2} - \alpha\right) - \beta\right) \\ &= \cos\left(\frac{\pi}{2} - \alpha\right) \cos \beta + \sin\left(\frac{\pi}{2} - \alpha\right) \sin \beta \\ &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{aligned}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\text{Proof of } \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

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$$\sin(\alpha - \beta) = \sin[\alpha + (-\beta)]$$

Rewrite as the sum of two angles.

$$\sin(\alpha - \beta) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta)$$

Use the sine of a sum formula

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta + \cos \alpha (-\sin \beta)$$

The cosine of a negative angle = cosine of the positive angle.

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

The sine of a negative angle = - the sine of the positive angle.

 Verify that $\sin(90^\circ - \theta) = \cos \theta$ when $\theta = 30^\circ$.

Do you recognize the cofunction formula?

 Verify the equation: $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{6}$

 Verify the equation: $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ for $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{6}$

 Evaluate $\sin \frac{7\pi}{12}$. Hint: use $\frac{\pi}{3}$ and $\frac{\pi}{4}$

$$\frac{1}{4}(\sqrt{2} + \sqrt{6})$$

 Evaluate $\sin \frac{5\pi}{12}$ Hint: use $\frac{\pi}{6}$ and $\frac{\pi}{4}$

Verify: $\sin(110^\circ - 50^\circ) = \sin 110^\circ \cos 50^\circ - \cos 110^\circ \sin 50^\circ$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2} \equiv \frac{\sqrt{3}}{2}$$

If $\tan \alpha = -\frac{4}{3}$ in Quadrant 2 and $\tan \beta = -\frac{5}{12}$ in Quadrant 4, evaluate each expression:

$\sin(\alpha + \beta)$ $\cos \beta$

$$\frac{63}{65}$$

$\cos(\alpha + \beta)$

$\cos \alpha$

Sum and Difference Formulas for the Tangent Function

Memorize these formulas

$$\left\{ \begin{array}{ll} \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} & \text{The tangent of a sum formula.} \\ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} & \text{The tangent of a difference formula.} \end{array} \right.$$

Verify: $\tan(\theta + \pi) = \tan \theta$. (This confirms that the tangent has a period of π .)

Simplify: $\tan(\theta + 3\pi)$ or verify $\tan(\theta + 3\pi) = \tan \theta$

If $\tan \alpha = -\frac{4}{3}$ in Quadrant 2 and $\tan \beta = \frac{15}{8}$ in Quadrant 3, find each of the following:

$$\sin(\alpha - \beta)$$

$$\cos(\alpha + \beta)$$

$$\tan(\alpha - \beta)$$

Prove the identity: $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot \theta$

Proof of the tangent of a sum and tangent of a difference formulas

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad \text{The tangent of a sum of angles}$$

Proof:

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$

divide the numerator and the denominator by $\cos \alpha \cos \beta$

$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

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$$\tan(\alpha + \beta) = \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{\frac{\cos \beta}{\cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad \text{Q.E.D.}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad \text{The tangent of a difference of angles}$$

Proof:

$$\begin{aligned} \tan(\alpha - \beta) &= \tan[\alpha + (-\beta)] \\ \tan(\alpha - \beta) &= \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)} \end{aligned}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan(\beta)}{1 - \tan \alpha (-\tan \beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad \text{Q.E.D.}$$

3.3.16 Double-angle and Power Reducing Formulas formulas

Memorize these

Double-angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta \quad \tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

Derive the formula for $\sin(2\theta)$

Power Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

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Derive the formulas for $\cos(2\theta)$

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Derive the formula for $\tan(2\theta)$

If $\sin \theta = \frac{4}{5}$, θ is in Quadrant I, find $\sin(2\theta)$

Write $\sin^4 \theta$ in terms of the first power.

Derive the formula for $\cos^2 \theta$

Derive the formula for $\tan^2 \theta$

Derive the formula for $\sin^2 \theta$

Use the power-reducing formula to find the exact value for $\sin(15^\circ)$.

$$\begin{aligned} \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} & \sin^2 \frac{\theta}{2} &= \frac{1 - \cos \theta}{2} & \sin^2 15^\circ &= \frac{1 - \cos 30^\circ}{2} & \sin 15^\circ &= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} \\ \sqrt{\sin^2 \theta} &= \sqrt{\frac{1 - \cos 2\theta}{2}} & \sqrt{\sin^2 \frac{\theta}{2}} &= \sqrt{\frac{1 - \cos \theta}{2}} & \sqrt{\sin^2 15^\circ} &= \sqrt{\frac{1 - \cos 30^\circ}{2}} & &= \sqrt{\frac{2 - \frac{\sqrt{3}}{2}}{2}} \\ \sin \theta &= \sqrt{\frac{1 - \cos 2\theta}{2}} & \sin \frac{\theta}{2} &= \sqrt{\frac{1 - \cos \theta}{2}} & \sin 15^\circ &= \sqrt{\frac{1 - \cos 30^\circ}{2}} & &= \sqrt{\frac{2 - \frac{\sqrt{3}}{2}}{2}} \\ & & & & \sin 15^\circ &= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} & &= \sqrt{\frac{2 - \frac{\sqrt{3}}{2}}{2}} \\ & & & & & & &= \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{4}} \\ & & & & & & & \sin 15^\circ = \frac{\sqrt{2 - \sqrt{3}}}{2} \end{aligned}$$

3.4 TRIGONOMETRY IDENTITIES

Reciprocal
Identities:

$$\begin{aligned} 1) \sin x &= \frac{1}{\csc x} & 2) \sec x &= \frac{1}{\cos x} & 3) \tan x &= \frac{1}{\cot x} \\ 4) \csc x &= \frac{1}{\sin x} & 5) \cos x &= \frac{1}{\sec x} & 6) \cot x &= \frac{1}{\tan x} \end{aligned}$$

Pythagorean

$$\text{Identities: } 7) \sin^2 x + \cos^2 x = 1 \quad 8) 1 + \tan^2 x = \sec^2 x \quad 9) 1 + \cot^2 x = \csc^2 x$$

Tangent and Cotangent Identities:

$$10) \tan x = \frac{\sin x}{\cos x} \quad 11) \cot x = \frac{\cos x}{\sin x}$$

Cofunction Identities:

$$12) \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad 13) \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad 14) \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Mejor morir de pie que
vivir en rodillas.

Toughness is in the soul and spirit, not in muscles.

Alex Karras

The first and greatest commandment is, Don't let them scare you.
Elmer Davis

Si se puede! Dolores Huerta

4.1.17 Solving Trigonometric Equations

Is $\theta = \frac{\pi}{3}$ a solution to the equation $\cos \theta = \frac{\sqrt{3}}{2}$? If not, what is the solution?

Solve the equation $\sin \theta = \frac{\sqrt{2}}{2}$ where $\theta = [0, 2\pi)$

Solve the equation $\sin 3x = 1$

$$x = \frac{\pi}{6} + \frac{2\pi k}{3}, \text{ where } k \text{ is an integer}$$

To solve trigonometric equations you will use the techniques you learned to solve algebraic equations such as: factoring, the quadratic formula, the zero-factor rule. However, before finding the values of θ , you will first need to solve the equation for the trig function, then determine the values of θ which satisfy the equation.

Solve the equation $2\sin\theta + \sqrt{2} = 0$, $\theta \in [0, 2\pi)$.

Hint: Solve for sine.

To solve trigonometric equations you will use the techniques you learned to solve algebraic equations such as: factoring, the quadratic formula, the zero-factor rule. However, before finding the values of θ , you will first need to solve the equation for the trig function, then determine the values of θ which satisfy the equation.

Solve $2\sin^2 x \cos x - \cos x = 0$, $x \in [0, 2\pi)$

Hint: factor out the common factor.

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$$\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \text{ and } \frac{7\pi}{4}$$

Solve: $2x^2 = 1 - x$

Solve: $2\sin^2 x = 1 - \sin x, \quad 0 \leq x < 2\pi$

Solve the equation $3\cos^2 x - 5\cos x - 4 = 0$

Hint: Use the quadratic formula.

Solve the quadratic trinomial equation: $4\sin^2 \theta = 6\sin \theta - 2 \quad 0 \leq \theta < 2\pi$ Hint: factor.

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

Solve using identities: $4\sin^2 \theta - 3 = \sin^2 \theta + \cos^2 \theta, \quad 0 \leq \theta < 2\pi$ Hint: Use Pythagorean Identity

Solve the equation $2\cos^2 x + 1 = -3\cos x$, where $0 \leq \theta < 2\pi$

$$x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$$

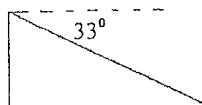
4.2.18 Using the Right Triangle and Trigonometry to Solve Problems

Determine the values of the six trigonometric functions of θ for a right triangle whose legs measure 5 and 12.

From a point 115 feet from the base of a tree, the angle of elevation to the top of the tree is 64.3° . What is the height of the tree rounded to the nearest foot? Draw and label the appropriate triangle.

239 feet

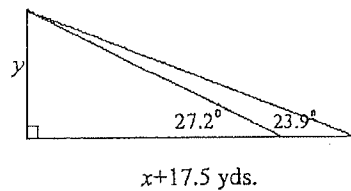
We can measure the distance from a plane to a radar station using sophisticated equipment. If the distance from a plane to a radar station is 160 miles and the angle of depression is 33° , determine the distance on the ground from a point directly below the plane to the radar station.



← 160 miles

The angle of elevation from a point on the ground to the top of a space shuttle is 27.2° . From a point 17.5 yards further away from the space shuttle, the angle of elevation is 23.9° . How tall is the space shuttle?

Hint: Find the tangent of both angles in terms of x and y , then solve the system of equations.



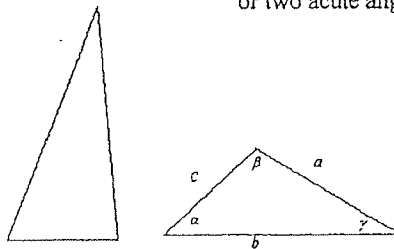
$$y \approx 56.3 \text{ yds} \approx 16 \text{ feet}$$

4.3.19 Law of Sines *Trigonometry Without a Right Triangle*

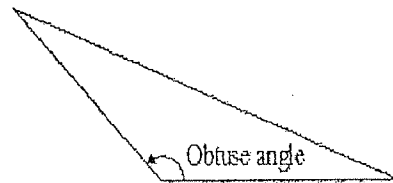
The Law of Sines and the Law of Cosines allow us to solve problems using a triangle which is oblique, that is, the triangle does not have a right angle.

You will “solve the oblique triangle” which means you will find the lengths of its sides and the angle measurements.

An oblique triangle has either three acute angles
or two acute angles and one obtuse angle (an angle between 90° and 180°).

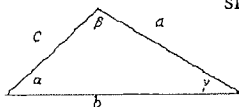


Three acute angles

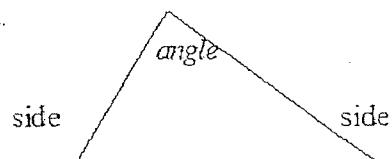


Two acute angles and one obtuse angle

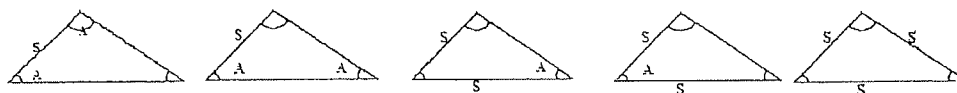
Label the oblique triangle so that side a is opposite angle α (alpha),
side b is opposite angle β (beta), and side c is opposite angle γ (gamma).



A triangle has six parts: 3 angles and 3 sides.



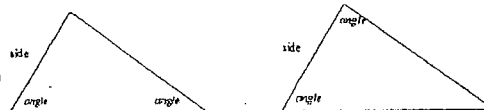
Label the triangle where Side Angle Side are known. Then label the others as indicated.
 SAS SSS ASA SAA SSA



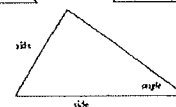
In order to solve an oblique triangle, you need to know the length of at least one side and any two other parts of the triangle—either two sides, two angles, or one angle and one side. Thus, there are four possibilities.

The first two cases are solved using the Law of Sines.

1. Two angles and any side (AAS or ASA)



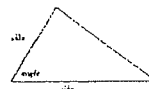
2. Two sides and the angle opposite one of the sides (SSA)



3. Three sides (SSS)



4. Two sides and their included angle (SAS)



The Law of Sines is introduced in this section and the Law of Cosines in the next section.

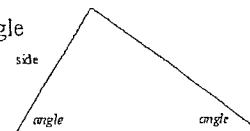
Thus, we will first consider oblique triangles where either two angles and a side are known or else two sides and an angle opposite one of the sides are known.

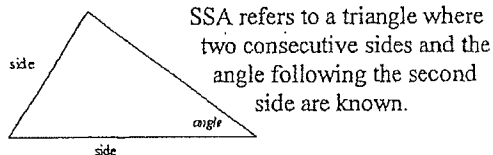
Use the **Law of Sines** to *solve* triangles where the known information is ASA or SAA or SSA.

ASA refers to a triangle where the known values are an angle, the side that follows it, and the angle at the other end of the known side.



SAA refers to a triangle where one side and the two consecutive angles following the side are known.



**Law of Sines**

For a triangle with sides a, b, c and opposite angles α, β, γ , *alpha, beta, gamma*, respectively,

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

or in the reciprocal form

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

ASA Triangle: Solve the triangle accurate to two decimal places:

$$\alpha = 43^\circ, \beta = 98^\circ, c = 22$$

$$\gamma = 39^\circ, a \approx 23.84 \text{ ft}, b \approx 34.62 \text{ ft}$$

AAS Triangle: Solve the triangle accurate to two decimal places:

$$\beta = 28.7^\circ, \gamma = 102.3^\circ, b = 27.4 \text{ ft}$$

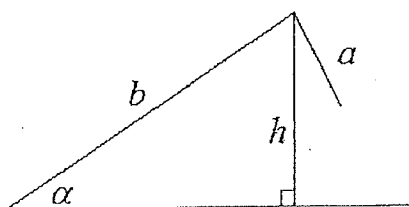
$$\alpha = 49^\circ, a \approx 43.06 \text{ ft}, c \approx 55.75 \text{ ft}.$$

SAA Triangle: Solve the triangle accurate to one decimal place:

$$\alpha = 22.5^\circ, \beta = 112.4^\circ, a = 16.3$$

$$\gamma = 45.1^\circ, b \approx 39.4, c \approx 30.2$$

SSA Triangle is known as The Ambiguous Case because the solution could result in no triangle, one triangle, or two triangles. The key to determining the number of triangles lies with the height & the fact that $h = b \sin \alpha$.

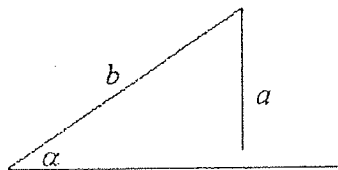


The ambiguous case refers to the fact that, when two sides and the angle opposite one of them are given, sometimes the Law of Sines cannot be used.

$$\sin \alpha = \frac{h}{b} \Rightarrow h = b \sin \alpha$$

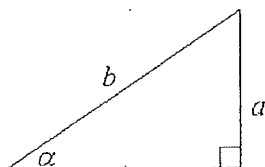
No Triangle

If $a < h = b \sin \alpha$, then there is no triangle since a doesn't reach the base.



One Right Triangle

If $a = h = b \sin \alpha$, then side a is just long enough to make one right triangle.



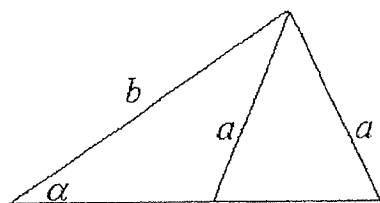
Two Oblique Triangles

or

One Oblique Triangle

If side a is shorter than side b
and side a is longer than h

if side $a \geq b$


 $a < b$ and $a > h$

 $a \geq b$

SSA Triangle: Solve the triangle: $a = 22, b = 12, \alpha = 42^\circ$

The Law of Sines produces one triangle.

$$\text{Solution: } \sin \beta \approx 0.43, \gamma = 154.6^\circ, c = \frac{3 \sin 114.6^\circ}{\sin 40^\circ} \approx 4.24$$

SSA Triangle: Find angle α if the triangle has the following measures $a = 42, b = 30, \alpha = 32^\circ$

The Law of Sines produces two triangles.

$$\alpha \approx 48^\circ \text{ or } 132^\circ$$

The Law of Sines shows there is no such triangle.

SSA Triangle: Find angle α : $a=15, c=20, \alpha=57^\circ$

The sine function cannot be larger than 1. Therefore, there is no such angle. Thus, there is no triangle with these measurements for a, c , and α .

4.4.20 Law of Cosines

We used the Law of Sines to solve oblique triangles where two angles were known (SAA or ASA) and the ambiguous case where SSA were known.

The Law of Cosines is used to solve oblique triangles where the three sides are known (SSS) or two sides and the included angle are known (SAS).

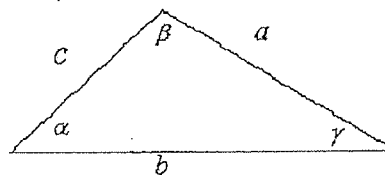
Law of Cosines Theorem

For a triangle with sides a, b, c and opposite angles α, β, γ respectively,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

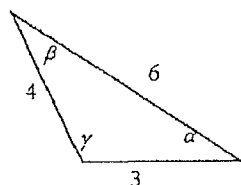
$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$



SAS Triangle Find the length of side b where $a=10, c=15, \beta=110^\circ$.

Solution: $b \approx 20.7$

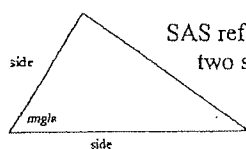
SSS Triangle Find the measure of angle β (to the nearest degree) where $a = 32$, $b = 20$, $c = 40$.



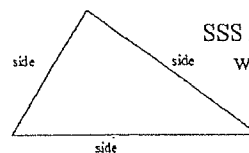
Solution: $\beta \approx 30^\circ$

Summary

Use the **Law of Cosines** to *solve* either SAS or SSS triangles.



SAS refers to a triangle where two sides and the included angle are known.



SSS refers to a triangle where all 3 sides are known.

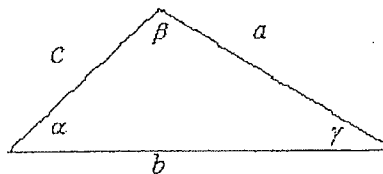
Law of Cosines Theorem

For a triangle with sides a , b , c and opposite angles α , β , γ respectively,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

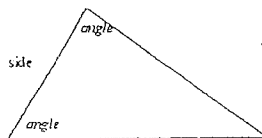
$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

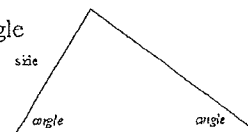


Use the **Law of Sines** to *solve* triangles where the known information is ASA or SAA or SSA.

ASA refers to a triangle where the known values are an angle, the side that follows it, and the angle



SAA refers to a triangle where one side and the two consecutive angles following



MAT182 Trigonometry
at the other end of the known side.

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the side are known.

Spring 2010

Law of Sines

For a triangle with sides a, b, c and opposite angles α, β, γ , *alpha, beta, gamma*, respectively,

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

or in the reciprocal form

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

The Greek Alphabet

Letter	Upper Case	Lower Case	Name	Letter	Upper Case	Lower Case	Name
A	A	α	alpha	O	O	o	omicron
B	B	β	beta		Ω	ω	omega
C	Γ	γ	gamma	P	Π	π	pi
	Ξ	ξ	xi		Φ	ϕ ϕ	phi
D	Δ	δ	delta		Ψ	ψ	psi
E	E	ε	epsilon	R	P	ρ	rho
	H	η	eta	sigma	S	Σ	ς
I	I	ι	iota	T	T	τ	tau
K	K	κ	kappa		Θ	θ	theta
L	Λ	λ	lambda	U	Υ	υ	upsilon
M	M	μ	mu	X	X	χ	chi
N	N	ν	nu	Z	Z	ζ	zeta

$$\pi = \text{pi} \approx 3.14159265 \text{ radians}$$

EXHIBIT 11

DECLARATION OF JOE SUEYOSHI

Joe Sueyoshi, being first duly sworn upon his oath, deposes and says:

1. I am over the age of eighteen and am competent to testify as to all the matters set forth herein and would so testify if called upon to do so.
2. I have personal knowledge of the matters set forth herein. My personal knowledge is based upon my observations and personal participation in the events described below.
3. I have been employed with Maricopa County Community College District ("MCCCD") at its Phoenix College ("PC") campus for the past 23 years as a Math Professor. In 2008, I was elected to serve as the Chair of the Math Department. Each Math Department Chair serves for a three year term. My second term is scheduled to run through Summer, 2014.
4. During the course of my 23-year career with MCCCD, I met and interacted with numerous faculty members. One such faculty member is Math Professor Dr. Cleopatria Martinez. Dr. Martinez also served as the Math Department Chair from 2002 through 2005.
5. In or around March of 2010, PC President Anna Solley notified me that Dr. Martinez had been failing to comply with copyright requirements. Specifically, Dr. Solley explained that Dr. Martinez had been copying math problems from copyright protected textbooks and inserting these problems into course materials

that she distributed to her students. Dr. Solley further explained that Dr. Martinez referred to these course materials as "Lecture Notes" and used them in lieu of a course textbook. Dr. Solley further informed me that MCCCCD administration had been trying to work with Dr. Martinez since January of 2010 to address their concerns regarded copyrighted materials. But as Dr. Solley explained, despite MCCCCD's repeated attempts to educate regarding copyright laws, Dr. Martinez continued to disregard the rights of copyright holders.

6. On April 2, 2010, Dr. Solley issued a directive revoking Dr. Martinez's copy privileges. The directive required Dr. Martinez to submit her copying requests to me for approval so that I could review her materials for possible copyright violations. A true and correct copy of the April 2, 2010 Directive is attached as Exhibit A. Dr. Solley told me that this procedure was put into effect because she had concluded that Dr. Martinez had exposed MCCCCD to potential legal liability for copyright violations.

7. Over the next several months, I followed Dr. Solley's instructions and reviewed Dr. Martinez's copy requests prior to submission to IKON, our on-site third-party copy service. If the materials Dr. Martinez provided appeared to be free of copyright concerns, I approved them. However, if I was able to identify portions of the materials that were copied from copyright protected textbooks, I denied the copy job. Similarly, if the copy job looked suspicious, I explained my concerns to

Dr. Martinez and allowed her to provide an explanation before I approved or denied the copy job. By way of example, at least one of Dr. Martinez's copy jobs included a series of math problems that were not consecutively numbered. As I can recall, the series of problems included only odd numbered problems and the numbering of the problem started in the 20s or 30s. Stated differently, the series of math problems were numbered in the following general manner (21, 23, 25, 27, 29 . . .). I found this to be extremely suspicious and asked Dr. Martinez why she had numbered the problems in this manner. Dr. Martinez, however, was unable to provide me with an explanation so I denied the copy job.

8. Although I repeatedly asked Dr. Martinez why she continually disregarded the rights of copyright holders, Dr. Martinez's only excuses were always that she "didn't know" that her actions potentially violated federal copyright laws and/or that she didn't see anything wrong with what she was doing.

9. Dr. Solley and I were hopeful that the suspension of Dr. Martinez's copy privileges would cause her to review her actions and respect the rights of copyright holders. However, Dr. Martinez continued to disregard copyright laws and actually tried to circumvent the April 2, 2010 Directive. For example, on or around April 19, 2010, Dr. Martinez attempted to bypass the April 2, 2010 Directive by having an adjunct math professor, Johnny Santellan, make 24 sets of copies of her "Lecture Notes" for distribution to her math students without obtaining my approval.

Math Department Secretary, Kelly Loucks, reported the incident involving Mr. Santellan to me shortly after it occurred.

10. I also personally witnessed repeated instances where Dr. Martinez attempted to bypass the approval process by printing multiple copies of her "Lecture Notes" directly to the math department laser printer.


11. In December 9, 2010, Dr. Solley issued a second directive that imposed further restrictions on Dr. Martinez's copying privileges. A true and correct copy of the December 9, 2010 Directive is attached as Exhibit B. Due to Dr. Martinez's ongoing refusal to comply with copyright laws, this directive is still in effect. The December 9, 2010 Directive prohibits Dr. Martinez from utilizing any course materials of her own creation. Instead, Dr. Martinez is required to use only course materials that are "approved by the math department" or that are "available in the bookstore for sale to students and that are authored by persons other than [Dr. Martinez]." The Directive further requires Dr. Martinez to submit photocopy jobs to me for approval. Dr. Solley informed me that she implemented these requirements and restrictions in order to prevent further misuse of copyrighted works by Dr. Martinez and to reduce MCCC'D's exposure to legal claims for copyright violations.

12. Finally, I have reviewed Dr. Martinez's Complaint and understand that Dr. Martinez is claiming that she was allegedly subjected to discrimination and harassment on the basis of her Mexican-American national origin and/or Hispanic

race. I never discriminated against Dr. Martinez or any other employee on the basis of their race, national origin, or any other protected characteristic. To the contrary, I treat employees consistently and expect them to follow the law as well as MCCCDC policies and procedures. For example, when one of Dr. Martinez's coworkers, Dmitri Logvinenko (Caucasian male of Russian descent), tried to use copyrighted course materials, I instructed him to: (1) immediately remove the materials from his course website; and (2) begin using Math Department approved course materials. A true and correct copy of my February 27, 2011 email to Mr. Logvinenko is attached as Exhibit C. The key difference between Dr. Martinez and Mr. Logvinenko was that Mr. Logvinenko apparently understood the severity of his actions, immediately complied with my request, and began using the department approved textbook for his course. Furthermore, while there are at least two other Hispanic/Mexican-American faculty members in the math department (Veronica Herrera and Maria Herrera-Bill), I do not subject them to the same copy restrictions as Dr. Martinez. This, of course, is because MCCCDC does not have reason to believe that either Ms. Herrera or Ms. Herrera-Bill have committed copyright violations.

Pursuant to the laws of the State of Arizona, I declare under penalty of perjury that the foregoing is true and correct.

Executed this 10th day of October, 2013 in Phoenix, Arizona.

A handwritten signature in cursive script, reading "Joe Sueyoshi", is written over a horizontal line.

JOE SUEYOSHI

Exhibit A



VIA HAND-DELIVERY

DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

April 2, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

Effective immediately, your copying privileges at the Phoenix College IKON Copy Center are suspended. - The recent incident involving the copying of portions of the book entitled "Precalculus Concepts Through Functions - A Right Triangle Approach to Trigonometry" by Sullivan and Sullivan gives me no other choice but to protect the College's legal interests by imposing this restriction. As detailed below, you knew or should have known that your actions violated federal law and Maricopa County Community College District ("District") regulations. Until I reinstate your copy privileges, all your copying requests must be approved in writing by your department chair in advance.

Respect for the work of others, including proper attribution of authorship and respect for copyright among other ethical standards, is a fundamental tenet of scholarship. I find it impossible to believe that a person who has been awarded a doctorate is so unfamiliar with the applicable legal and ethical standards. Moreover, during your long career with Phoenix College and the Maricopa County Community College District, you received extensive information about District Administrative Regulation 3.2 concerning copyright law and fair use guidelines as well as information about the federal Copyright Act. Administrative Regulation 3.2, which may be found on page 42 of the Blue Book, which is prominently available on the Legal Services Department's website as are the District's Administrative Regulations, of which that regulation is a part. All District employees were e-mailed a notice of the availability of the Blue Book on January 6, 2009 and asked to bookmark and acknowledge it. Administrative Regulation 3.2 reads as follows:

3.2 Copyright Regulation

1. *It is the intent of the Governing Board of the Maricopa County Community College District to adhere to the provisions of the U.S. Copyright Law (Title 17, United States Code Section 101 et seq.). Though there continues to be controversy regarding interpretation of the Copyright Law, this policy represents a sincere effort by the Board to operate legally within the District.*
2. *The Governing Board directs the Chancellor or his designee(s) to develop and distribute to employees guidelines that (1) clearly discourage violation of the Copyright Law and (2) inform employees of their rights and responsibilities under the Copyright Law.*
3. *Each college president or provost and the Chancellor shall name an individual(s) at each district location who will assume the responsibilities of distributing copyright guidelines, act as a resource person regarding copyright matter and provide training programs on current copyright laws.*

MAIN CAMPUS
1202 West Thomas Road
Phoenix, AZ 85013
(602) 285-7433

PC DOWNTOWN CAMPUS
640 North 1st Avenue
Phoenix, AZ 85003
(602) 223-4000

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phoenixcollege.edu
EEOC000191
A Maricopa Community College.

Dr. Cleopatria Martinez

April 2, 2010

Page 2

4. *Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) Licenses or contractual agreements, or (4) other permission.*
5. *The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.*
6. *In order to assist employees and students in complying with the Copyright Law, appropriate notices shall be placed on or near all equipment capable of duplicating copyrighted materials.*

Moreover, not long before the recent incident arose, Phoenix College officials including Ronnie Elliott, Paul DeRose, and Casandra Kakar as well as District legal counsel Maggie McConnell extensively discussed with you the requirements of the Administrative Regulation. Finally, Phoenix College officials encouraged you to attend the March 1, 2010 Phoenix College workshop to learn more about copyright/fair use. However, you chose not to attend the workshop.

You are reminded that Section 4 of Administrative Regulation 3.2, quoted above, clearly advises that the Governing Board prohibits and disapproves of unauthorized duplication in any form. Section 5 of the Regulation further advises that employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

You are finally admonished that any further copyright/fair use violations will lead to disciplinary action being taken against you, up to and including termination of your employment.

I am willing to arrange for individual training and consulting on copyright if you feel it would be helpful. But I will restore your copying privileges only when I determine that you not only understand your legal obligations, but also that you will strictly adhere to them.

Sincerely,



Anna Solley, Ed.D.
President

cc: Mr. Paul DeRose
Mr. Richard Galvan
Ms. Casandra Kakar
Ms. Maggie McConnell

EEOC000192

MCCCD/Martinez 01406

Exhibit B



VIA HANDDELIVERY

DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

December 9, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

During the spring semester of 2010, I initiated an administrative evaluation based on Mr. Joe Sueyoshi's complaint, pursuant to section 3.7 of the Residential Faculty Policies Manual. By agreement, the Evaluation Team did not undertake its task until this fall semester.

I have reviewed the report of the Evaluation Team that was sent to me on November 17, 2010. The Evaluation Team which was advised by expert counsel, found that your work violated copyright. In addition, the Evaluation Team found that you were insubordinate and failed to follow District Legal Counsel's and my instructions and continued to request and persist in printing or copying unauthorized materials. The Evaluation Team agreed "that the administration should take appropriate disciplinary action following district policies in regards to this matter."

I have also considered our experiences under the current temporary arrangement in which Mr. Sueyoshi reviews materials you submit and may approve copying if they appear to be free of copyright and attribution problems. Given the Evaluation Team's findings and my lack of confidence in your willingness to follow the law as well as Maricopa policies and procedures due to your repeated copyright violations, I no longer consider the arrangement to be sufficient to protect the institution's legal and financial interests.

Pursuant to section 3.7.4 of the RFP, I now direct you to use only course materials approved by the department, that are available in the bookstore for sale to the students and that are authored by persons other than yourself. With department chair and Vice President for Academic Affairs approval, you may customize these materials for your students using traditional course packs or new online programs such as McGraw-Hill's "Create" (<http://create.mcgraw-hill.com/createonline/index.html#>) or McMillan's "Dynamic Books" (<http://dynamicbooks.vitalbook.com/>). All materials you use must be clearly attributed to authors other than yourself, and you must produce evidence of purchase or the author's permission to use them (e.g., a copy of the letter giving you a courtesy copy of the instructor's edition). If you wish to make copies of or from these materials, you must first produce written permission from the author and/or publisher to Mr. Sueyoshi. In addition, requests for the copying of these materials or your syllabi and quizzes/tests must be submitted in a timely manner (minimum of two work days notice) to Mr. Sueyoshi for approval.

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MCCCD/Martinez01055

MCCCD/Martinez 01408

Dr. Cleopatria Martinez
December 9, 2010
Page 2

This direction is intended to communicate job duties to you within the meaning of Governing Board employment standard A4.3. In accordance with that section of the All Employee Policy Manual, willful and intentional violation of these instructions will be considered grounds for disciplinary action, up to and including termination of your employment.

Sincerely,

A handwritten signature in cursive script, appearing to read "Anna Solley", with a long horizontal flourish extending to the right.

Anna Solley, Ed.D.
President

cc: Mr. Lee Combs
Ms. Casandra Kakar
Mr. Joe Sueyoshi

Exhibit C

6/25/13

Maricopa Community College District Mail - Unauthorized Textbook Materials?



Unauthorized Textbook Materials?

Joe Sueyoshi <joe.sueyoshi@pcmail.maricopa.edu>
To: Dmitri Logvinenko <dmitri.logvinenko@pcmail.maricopa.edu>
Cc: Kelly Loucks <kelly.loucks@pcmail.maricopa.edu>

Sun, Feb 27, 2011 at 8:15 PM

Dmitri,

I haven't been able to reach you face to face, but perhaps it's better that this discussion is documented. I have 3 immediate questions:

- (1) Are you using the Briggs-Cochran calculus text for your MAT220 class?
- (2) If not, are the MAT220 materials that are available on your webpage of your own creation or from another source?
- (3) Did you create the powerpoint slides contained in your MAT082 resources or are they from the Bittinger-Penna text?

(1 & 2) If you are not using the Briggs-Cochran text, this would be a violation of our department's textbook adoption policies. Since you previously did this with a MAT213 course, it appears that you do not want to follow department policies concerning textbooks, even though one of the reasons we have such policies is to protect the department faculty. I only found out (indirectly) from one of your MAT220 students last week and was directed to your webpage. It appears this is the case, but I hope I am wrong.

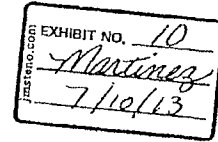
(3) If the powerpoint slides are resources from the Bittinger-Penna text, they cannot be placed on a webpage open to the public. I believe that would be a copyright violation unless you received permission from the publisher. Do you realize the district has already been fined \$150,000 for a similar incident on another campus. I don't know what punishment the instructor received, but it can't be good. All such copyrighted materials should be removed **immediately** from your webpage.

If you think this is not a major concern, did you know that one of the faculty members in our own department is receiving on-going disciplinary action for copyright violations which could still potentially lead to termination?

Sorry if you feel strongly otherwise, but I'm just trying to protect you. I'm doing my job as a department chair.

joe

EXHIBIT 12



Subject: Printing of Course Material
 From: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>
 Date: Tue, 26 Jan 2010 07:52:16 -0700
 To: Cleopatria Martinez <cleopatria.martinez@pcmail.maricopa.edu>
 CC: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>, Lee Combs <lee.combs@domain.maricopa.edu>, Margaret McConnell <margaret.mcconnell@domain.maricopa.edu>, Anna Solley <anna.solley@pcmail.maricopa.edu>, Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Cleopatria,

In order to respond to your email dated January 20, 2010, I solicited the guidance of our district legal counsel, Margaret McConnell, regarding copyright infringement and fair use as defined in federal law. I did provide Margaret with copies of your requests and a copy of the adopted text book, and this morning received her findings based on the federal copyright law, including the limitations of fair use.

The District's Administrative Regulation 3.2 provides in part as follows:

4. Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) licenses or contractual agreements, or (4) other permission.
5. The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

The focus here is whether your use of certain materials meets one of the criteria specified in Para. 4 quoted above. We don't have any information that suggests that your use fits within the specific exception under the copyright law authorizing the use of copyrighted materials without permission. Nor have we seen any licenses or written permission from the copyright holder, Pearson, of the materials for use of their materials in your lecture notes. Therefore, we must focus on "fair use."

The doctrine of fair use allows for the use of copyrighted works without the owner's permission. It protects limited uses of copyrighted works from being an infringement. The doctrine is, however, determined on a case-by-case basis.

The federal Copyright Act provides that the "fair use of a copyrighted work for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright." Again, not all educational uses are fair use. Four factors are considered in determining whether the use of a work is a fair use:

1. the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;
2. the nature of the copyrighted work;
3. the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and
4. the effect of the use upon the potential market for or value of the copyrighted work.

17 U.S.C. § 107. Courts decide fair use by weighing each factor individually and then determining which way the combination of factors tips.

In light of those legal standards, we

have reviewed your January 20 e-mail and the following materials - a document entitled "Phoenix College Fall 2009, MAT 182 Trigonometry Section, Lecture Notes, Dr. Cleopatria Martinez" and a book, Precalculus: Concepts Through Functions: A Right Triangle Approach to Trigonometry, Instructor's Edition, authored by Michael Sullivan and Michael Sullivan, III. The book indicates that the copyright holder is Pearson Education, Inc. and it says: "All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher."

The first issue here is whether your lecture notes are original works or problems created by you, or whether portions are instead copied from the Precalculus book or some other copyrighted source

In reviewing parts of your notes and the book, we have some real concerns. Attached are some pages of your notes on which we have

handwritten the page numbers of the Preetz us book as the source of the example, text or problem. Once we came up with these 10 or so correlations, we stopped looking, as the problem seemed clear. The examples or problems are copied verbatim into the lecture notes.

Additionally, there is no attribution in each of these instances of the source of the example or problem, that is, no identification of Pearson as the copyright holder. Whether you intend to or not, the lecture notes give the distinct impression that they are your original work, which is incorrect. That lack of attribution is, by itself, a legal issue. As to what the breadth of the problem is, only you know that because we don't know all of the resources that you may have used for those notes.

Moreover, it is our understanding that you may have been using these notes over a period of time.

As you can see from the listing of the "fair use" requirements, the use must not be one that, if it became widespread over an entire market, would reduce the potential market for the copyrighted work. In this case, copying problem or examples from the book, with the potential that the students don't need to purchase it, would make your use without permission a problem.

Other guidelines developed under the auspices of the federal government specify that use must "spontaneous." It is hard to make a case that this is spontaneous use if you use the materials over and over each semester. In any event, it would appear to have taken a significant effort to copy the problems and examples into the text of your lecture notes, thereby eliminating any "spontaneity."

Based on the above, we believe that you need to get written permission to use portions of the book in your lecture notes from Pearson, and make sure that you note that Pearson is the copyright holder of materials. Publishers such as Pearson have specific staff dedicated to this issue. You should find out who those are, identify the portions of the book that you wish to use, and obtain permission in writing.

Bottom line, Cleopatria, you must secure written permission from the publisher before we are able to print your material.

Regards
Ronnie

Lecture Notes and Book.pdf Content-Type: application/pdf
Content-Encoding: base64

EXHIBIT 13

Case 2:12-cv-00702-DGC Document 68-9 Filed 10/11/13 Page 115 of 122

Subject: Conversation

From: Margaret McConnell <margaret.mcconnell@domail.maricopa.edu>

Date: Thu, 28 Jan 2010 16:28:22 -0700

To: Cleopatria Martinez <cleopatria.martinez@pemail.maricopa.edu>

CC: Ronnie Elliott <ronnie.elliott@pemail.maricopa.edu>, Casandra Kakar <casandra.kakar@pemail.maricopa.edu>, Lee Combs <lee.combs@domail.maricopa.edu>, Anna Solley <anna.solley@pemail.maricopa.edu>

Cleopatria -

As we discussed today, it is inappropriate to copy anything from copyrighted source and reproduce it without the written permission of the copyright holder. It is made more egregious if a person reproduces it without providing notice on that material about who owns the copyright.

I referred you to the information about copyright law on the Legal Services Department's website (since 2003). MCCC's copyright guidelines and other resources are available there. The site is: <http://www.maricopa.edu/legal/ip/index.htm>

Over the last four years, I have personally participated in two copyright seminars for MCCC faculty sponsored by MCCC's MCLI. There are also excellent copyright resources for faculty on the MCLI website at: <http://mcli.maricopa.edu/copyright>

I explained to you today that I represent the District, and not any individual employee, so I can't give you personal advice. However, I want emphasize the importance of digesting the copyright information available and following it. A violation of copyright risks the assessment of potentially substantial fines under copyright law, and of employee sanctions under the District's copyright policy.

Maggie



Margaret E. McConnell
Assistant General Counsel



Office of General Counsel
3411 West 14th Street
Tempe, AZ 85281-6942
Tel: 480.731.8888 Fax: 480.731.8990
www.maricopa.edu/legal

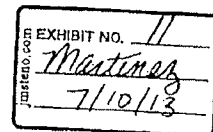


EXHIBIT 14

Meeting Summary from 2/5/10

Subject: Meeting Summary from 2/5/10
From: Paul DeRose <paul.derose@pcmail.maricopa.edu>
Date: Fri, 12 Feb 2010 20:58:42 -0700
To: Cleopatria Martinez <cleopatria.martinez@pcmail.maricopa.edu>
CC: Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Listed below is a summary of the meeting from 2/5/10.

Present: Casandra Kakar, Paul DeRose, Cleopatria Martinez, Denny Sheeshan

Summary: Copyright laws were discussed. The copyright verbiage is printed in the front of the textbook that is being used in the Math class. In order to use material from the text, prior approval must be obtained from the Publisher's Legal department. An approval from the Publisher's rep is not acceptable. Cleopatria was referred to the email sent by Maggie McConnell which describes the copyright issue and contains a link to seek Publisher approval. Cleopatria said she understands the copyright law and will not use materials from any textbook without prior approval from the Publisher's Legal office. If Cleopatria decides to bind her notes/handouts, Casandra recommends that they be distributed by the bookstore - similar to the practice used by other faculty in her department and at Phoenix College. This will be a nominal fee to the students, and will also minimize costs to the department's print budget. The posting of notes on Blackboard is also an option.

In addition to our discussion, we would also like to encourage you to attend an upcoming Copyright Workshop that is being held at Phoenix College. This workshop is scheduled for March 1st from 1:00pm - 3:00pm.

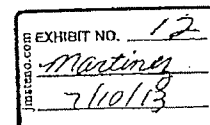


EXHIBIT 15

DECLARATION OF ANNA SOLLEY

Anna Solley, being first duly sworn upon her oath, deposes and says:

1. I am over the age of eighteen and am competent to testify as to all the matters set forth herein and would so testify if called upon to do so.

2. I have personal knowledge of the matters set forth herein. My personal knowledge is based upon my observations and personal participation in the events described below.

3. I have been employed by Maricopa County Community College District ("MCCCD") as the President of Phoenix College ("PC") for more than eight years. PC is a taxpayer supported entity that relies on funding and appropriations from the Arizona legislature. In my capacity as President of Phoenix College, I have met and interacted with numerous faculty members. One such faculty member was Math Professor Dr. Cleopatria Martinez.

4. I have reviewed Dr. Martinez's Complaint and understand that Dr. Martinez is claiming that she was allegedly subjected to discrimination and harassment on the basis of her Mexican-American national origin and/or Hispanic race. As discussed below, I was the decision maker for many of the actions which Dr. Martinez incorrectly attributes to discrimination or harassment on the basis of her race or national origin. I am of the same national origin (Mexican-American) and race (Hispanic) as Dr. Martinez. In fact, I am a long time activist for the civil rights of Hispanics; am an active member of numerous Hispanic organizations including, but not limited to, National Community College Hispanic Council, the Raul H. Castro Institute Advisory Committee, Superintendent John Huppenthal's Hispanic Advisory Group, and the Lumina Foundation Funded Initiative with Degree Phoenix; was born in Nogales, Arizona; and speak fluent Spanish.

5. To be perfectly clear, I never discriminated against Dr. Martinez or any other employee on the basis of their race, national origin, or any other protected characteristic. To the contrary, any actions I took against Dr. Martinez were the direct

result of her refusal to respect the rights of copyright holders and her unreasonable exposure of MCCCDC to legal liability for a copyright infringement lawsuit.

6. In January of 2010, Phoenix College Vice President of Administrative Services Ronnie Elliott and Vice President of Academic Affairs Casandra Kakar informed me that Dr. Martinez had submitted a set of suspicious copy requests to IKON, our on-site copy service. Ms. Elliott and Dr. Kakar further advised me that Dr. Martinez had asked IKON to copy approximately 30 sets of a 73 page document titled "2009 MAT 182," as well as a similar document for her MAT 187 mathematics course which she was responsible for teaching. Ms. Elliott and Dr. Kakar also told me that IKON had expressed concern that the documents might contain math problems taken directly from copyrighted textbooks and had contacted Ms. Elliott for guidance on whether to process the copy requests. Ms. Elliott, in turn, sought the guidance and input of Ms. Kakar and myself.

7. After reviewing the materials and consulting with MCCCDC's General Counsel, Lee Combs, I decided that allowing the materials to be photocopied and distributed to students could expose MCCCDC to liability for copyright infringement. As a result, on or around January 12, 2010, I instructed Ms. Elliott send an email to Dr. Martinez explaining that MCCCDC had learned that the Fall 2009 and Spring 2010 materials she submitted to IKON had been reviewed and potentially infringed on the rights of copyright holders. A true and correct copy of Ms. Elliott's January 12, 2010 email to Dr. Martinez is attached as Exhibit A.

8. Shortly after Ms. Elliott sent the January 12, 2010 email, Ms. Elliott informed me that Dr. Martinez had started asking Ms. Elliott if she could "fix" the copyright issues by changing one or two numbers from the problems that were copied directly from copyright protected textbooks. In fact, based on my own discussions with Dr. Martinez, I learned that Dr. Martinez had a practice of copying math problems from copyright protected textbooks and inserting these problems into course materials that she distributed to her students. Dr. Martinez referred to these course materials as "Lecture

Notes.” As Dr. Martinez indicated in her October 15, 2010 email to me, Dr. Martinez used her “Lecture Notes” in lieu of a course textbook. Ms. Elliott informed me that Dr. Martinez’s attempts to elicit advice from her (and others) on how to evade applicable copyright laws made Ms. Elliott extremely uncomfortable. Accordingly, I instructed Ms. Elliott to confer with MCCCCD’s district counsel, and then follow up with Dr. Martinez regarding compliance with copyright requirements.

9. On January 26, 2010, Ms. Elliott sent Dr. Martinez another email outlining MCCCCD’s concerns regarding her suspected copyright violations. Specifically, Ms. Elliott explained that she had solicited the advice of district counsel, Margaret McConnell, regarding copyright infringement and fair use. Ms. Elliott further explained that we were concerned that Dr. Martinez’s actions could subject MCCCCD to a copyright infringement claim because we were able to identify at least 10 instances where problems had been copied directly from the copyrighted Precalculus textbook authored by Sullivan and Sullivan. A true and correct copy of Ms. Elliott’s January 26, 2010 email to Dr. Martinez is attached as Exhibit B.

10. Following Ms. Elliott’s emails, various additional members of MCCCCD administration communicated their concerns to Dr. Martinez regarding her misuse of copyrighted materials. Specifically, on January 28, 2010, Ms. McConnell separately discussed copyright issues with Dr. Martinez over the phone. Moreover, on February 5, 2010, Dr. Kakar and Vice President of Administrative Services Paul DeRose met with Dr. Martinez to discuss Dr. Martinez’s failure to comply with copyright requirements. Ms. McConnell, Dr. Kakar, and Mr. DeRose informed me about their conversations with Dr. Martinez shortly after they occurred.

11. On March 1, 2010, MCCCCD held a copyright workshop that district counsel Maggie McConnell instructed. Ms. McConnell informed me that Dr. Martinez was not present at the workshop. Since Dr. Martinez did not attend the March 1, 2010 workshop, I made arrangements for PC Librarian Ann Roselle to separately meet with Dr. Martinez and conduct a personalized one-on-one copyright training session with her.

This one-on-one training session occurred on April 15, 2010. Unfortunately, however, despite these repeated attempts by MCCC'D administration to educate Dr. Martinez regarding copyright laws, Dr. Martinez continued to disregard the rights of copyright holders.

12. Normally, a professor at PC is allowed to make a request to photocopying of materials for instruction directly to PC's on-campus copy service, IKON. However, because of Dr. Martinez's misuse of copyrighted materials, and after seeking the advice of MCCC'D's General Counsel, Lee Combs, I decided to impose restrictions on Dr. Martinez's copying privileges. On April 2, 2010, I issued a directive revoking Dr. Martinez's copy privileges. The directive required Dr. Martinez to submit her copying requests to Math Department Chair Joe Sueyoshi for approval so that he could review her materials for possible copyright violations. A true and correct copy of my April 2, 2010 Directive is attached as Exhibit C. This procedure was put into effect because I concluded that Dr. Martinez had exposed MCCC'D to potential legal liability for copyright violations.

13. I was hopeful that the suspension of Dr. Martinez's copy privileges would cause her to review her actions and respect the rights of copyright holders. However, Dr. Martinez continued to disregard copyright laws and actually tried to circumvent my April 2, 2010 Directive. For example, on or around April 19, 2010, Dr. Martinez attempted to bypass the copy restrictions by having an adjunct math professor, Johnny Santellan, make 24 sets of copies of her "Lecture Notes" for distribution to her math students. There were also repeated instances where Dr. Martinez bypassed the approval process by printing multiple copies of her "Lecture Notes" directly to the math department laser printer. Math Department Chair Joe Sueyoshi reported both of the incidents to me shortly after they occurred.

14. In or around the fall of 2010, MCCC'D's General Counsel, Lee Combs, informed me that he was seeking a legal opinion from an outside copyright expert, Sean Garrison, regarding MCCC'D's potential legal liability for copyright infringement

resulting from Dr. Martinez's actions. After Mr. Garrison reviewed several samples of Dr. Martinez's "Lecture Notes," he prepared a report summarizing his findings. The report, which I reviewed, confirms that "significant portions of [the] Course Materials have been copied from other [copyrighted] sources" such as Basic Mathematics by Charles P. McKeague (published by Cengage Education); Prealgebra by Alan R. Tussy and R. David Gustafson (published by Thomson Learning, Inc.); and Precalculus by Michael Sullivan and Michael Sullivan, III (published by Pearson Education, Inc.). A true and correct copy of the report is attached as Exhibit D. Mr. Garrison further advised MCCCDC to prohibit Dr. Martinez from further distribution and use of her "Lecture Notes" because they presented a "significant risk of [copyright] infringement." Mr. Combs informed me that he concurred with Mr. Garrison's findings.

15. Based on the advice and conclusions of Mr. Garrison and Mr. Combs, I issued a December 9, 2010 Directive that imposed further restrictions on Dr. Martinez's copying privileges. A true and correct copy of my December 9, 2010 Directive is attached as Exhibit E. Due to Dr. Martinez's ongoing refusal to comply with copyright laws, this directive is still in effect. The December 9, 2010 Directive prohibits Dr. Martinez from utilizing any course materials of her own creation. Instead, Dr. Martinez is required to use only course materials that are "approved by the math department" or that are "available in the bookstore for sale to students and that are authored by persons other than [Dr. Martinez]." The Directive further requires Dr. Martinez to submit photocopy jobs to the Math Department Chair for his approval. I implemented these requirements and restrictions in order to prevent further misuse of copyrighted works by Dr. Martinez and to reduce MCCCDC's exposure to legal claims for copyright violations.

Pursuant to the laws of the State of Arizona, I declare under penalty of perjury that the foregoing is true and correct.

Executed this 10th day of October, 2013 in Phoenix, Arizona.



ANNA SOLLEY

Exhibit A

Case 2:12-cv-00702-DGC Document 68-2 Filed 10/11/13 Page 87 of 118

[Fwd: [Fwd: Re: Printing of Course Material]]

Subject: [Fwd: [Fwd: Re: Printing of Course Material]]
From: Paul DeRose <paul.derose@pcmail.maricopa.edu>
Date: Thu, 02 Sep 2010 15:29:11 -0700
To: Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Subject: Re: Printing of Course Material
From: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>
Date: Tue, 12 Jan 2010 14:14:21 -0700
To: "cleopatria.martinez" <cleopatria.martinez@pcmail.maricopa.edu>

MAT 182 and 187 are the two I'm aware of at this point.

cleopatria.martinez wrote:

Please remind me, what are the items I had printed for fall that were suspect and the same for my spring requests.

Ronnie Elliott wrote:

Cleopatria,

As you may or may not know, we have mechanisms in place to red flag potential copy right issues. Last week it was brought my attention that some of the items you had printed for fall were suspect and the same for your spring requests. Do you have any document of approval from the publisher to use their material? If so, would you please provide that documentation to me so that I can release your requests to be printed. If you do not have such approval, then we will not be able to print your requests at this time.

Ronnie

Re: Printing of Course Material.eml

Content-Type: message/rfc822
Content-Encoding: 7bit

Exhibit B

Subject: Printing of Course Material
 From: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>
 Date: Tue, 26 Jan 2010 07:52:16 -0700
 To: Cleopatria Martinez <cleopatria.martinez@pcmail.maricopa.edu>
 CC: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>, Lee Combs <lee.combs@domail.maricopa.edu>, Margaret McConnell <margaret.mcconnell@domail.maricopa.edu>, Anna Solley <anna.solley@pcmail.maricopa.edu>, Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Cleopatria,

In order to respond to your email dated January 20, 2010, I solicited the guidance of our district legal counsel, Margaret McConnell, regarding copyright infringement and fair use as defined in federal law. I did provide Margaret with copies of your requests and a copy of the adopted text book, and this morning received her findings based on the federal copyright law, including the limitations of fair use.

The District's Administrative Regulation 3.2 provides in part as follows:

4. Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) licenses or contractual agreements, or (4) other permission.

5. The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

The focus here is whether your use of certain materials meets one of the criteria specified in Para. 4 quoted above. We don't have any information that suggests that your use fits within the specific exception under the copyright law authorizing the use of copyrighted materials without permission. Nor have we seen any licenses or written permission from the copyright holder, Pearson, of the materials for use of their materials in your lecture notes. Therefore, we must focus on "fair use."

The doctrine of fair use allows for the use of copyrighted works without the owner's permission. It protects limited uses of copyrighted works from being an infringement. The doctrine is, however, determined on a case-by-case basis.

The federal Copyright Act provides that the "fair use of a copyrighted work for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright." Again, not all educational uses are fair use. Four factors are considered in determining whether the use of a work is a fair use:

1. the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;
2. the nature of the copyrighted work;
3. the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and
4. the effect of the use upon the potential market for or value of the copyrighted work.

17 U.S.C. § 107. Courts decide fair use by weighing each factor individually and then determining which way the combination of factors tips.

In light of those legal standards, we

have reviewed your January 20 e-mail and the following materials - a document entitled "Phoenix College Fall 2009, MAT 182 Trigonometry Section, Lecture Notes, Dr. Cleopatria Martinez" and a book, Precalculus: Concepts Through Functions: A Right Triangle Approach to Trigonometry, Instructor's Edition, authored by Michael Sullivan and Michael Sullivan, III. The book indicates that the copyright holder is Pearson Education, Inc. and it says: "All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher."

The first issue here is whether your lecture notes are original works or problems created by you, or whether portions are instead copied from the Precalculus book or some other copyrighted source

In reviewing parts of your notes and the book, we have some real concerns. Attached are some pages of your notes on which we have

handwritten the page numbers of the Precalculus book as the source of the example, text or problem. Once we came up with these 10 or so correlations, we stopped looking, as the problem seemed clear. The examples or problems are copied verbatim into the lecture notes.

Additionally, there is no attribution in each of these instances of the source of the example or problem, that is, no identification of Pearson as the copyright holder. Whether you intend to or not, the lecture notes give the distinct impression that they are your original work, which is incorrect. That lack of attribution is, by itself, a legal issue. As to what the breadth of the problem is, only you know that because we don't know all of the resources that you may have used for those notes.

Moreover, it is our understanding that you may have been using these notes over a period of time.

As you can see from the listing of the "fair use" requirements, the use must not be one that, if it became widespread over an entire market, would reduce the potential market for the copyrighted work. In this case, copying problem or examples from the book, with the potential that the students don't need to purchase it, would make your use without permission a problem.

Other guidelines developed under the auspices of the federal government specify that use must "spontaneous." It is hard to make a case that this is spontaneous use if you use the materials over and over each semester. In any event, it would appear to have taken a significant effort to copy the problems and examples into the text of your lecture notes, thereby eliminating any "spontaneity."

Based on the above, we believe that you need to get written permission to use portions of the book in your lecture notes from Pearson, and make sure that you note that Pearson is the copyright holder of materials. Publishers such as Pearson have specific staff dedicated to this issue. You should find out who those are, identify the portions of the book that you wish to use, and obtain permission in writing.

Bottom line, Cleopatria, you must secure written permission from the publisher before we are able to print your material.

Regards
Ronnie

Lecture Notes and Book.pdf Content-Type: application/pdf
Content-Encoding: base64

Exhibit C



VIA HAND-DELIVERY

DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

April 2, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

Effective immediately, your copying privileges at the Phoenix College IKON Copy Center are suspended. The recent incident involving the copying of portions of the book entitled "Precalculus Concepts Through Functions – A Right Triangle Approach to Trigonometry" by Sullivan and Sullivan gives me no other choice but to protect the College's legal interests by imposing this restriction. As detailed below, you knew or should have known that your actions violated federal law and Maricopa County Community College District ("District") regulations. Until I reinstate your copy privileges, all your copying requests must be approved in writing by your department chair in advance.

Respect for the work of others, including proper attribution of authorship and respect for copyright among other ethical standards, is a fundamental tenet of scholarship. I find it impossible to believe that a person who has been awarded a doctorate is so unfamiliar with the applicable legal and ethical standards. Moreover, during your long career with Phoenix College and the Maricopa County Community College District, you received extensive information about District Administrative Regulation 3.2 concerning copyright law and fair use guidelines as well as information about the federal Copyright Act. Administrative Regulation 3.2, which may be found on page 42 of the Blue Book, which is prominently available on the Legal Services Department's website as are the District's Administrative Regulations, of which that regulation is a part. All District employees were e-mailed a notice of the availability of the Blue Book on January 6, 2009 and asked to bookmark and acknowledge it. Administrative Regulation 3.2 reads as follows:

3.2 Copyright Regulation

1. *It is the intent of the Governing Board of the Maricopa County Community College District to adhere to the provisions of the U.S. Copyright Law (Title 17, United States Code Section 101 et seq.). Though there continues to be controversy regarding interpretation of the Copyright Law, this policy represents a sincere effort by the Board to operate legally within the District.*
2. *The Governing Board directs the Chancellor or his designee(s) to develop and distribute to employees guidelines that (1) clearly discourage violation of the Copyright Law and (2) inform employees of their rights and responsibilities under the Copyright Law.*
3. *Each college president or provost and the Chancellor shall name an individual(s) at each district location who will assume the responsibilities of distributing copyright guidelines, act as a resource person regarding copyright matter and provide training programs on current copyright laws.*

MAIN CAMPUS
1202 West Thomas Road
Phoenix, AZ 85013
(602) 285-7433

PC DOWNTOWN CAMPUS
640 North 1st Avenue
Phoenix, AZ 85003
(602) 223-4000

GO FAR. CLOSE TO HOME.

phoenixcollege.edu

EEOC000191

A Maricopa Community College.

Dr. Cleopatria Martinez

April 2, 2010

Page 2

4. *Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) Licenses or contractual agreements, or (4) other permission.*
5. *The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.*
6. *In order to assist employees and students in complying with the Copyright Law, appropriate notices shall be placed on or near all equipment capable of duplicating copyrighted materials.*

Moreover, not long before the recent incident arose, Phoenix College officials including Ronnie Elliott, Paul DeRose, and Casandra Kakar as well as District legal counsel Maggie McConnell extensively discussed with you the requirements of the Administrative Regulation. Finally, Phoenix College officials encouraged you to attend the March 1, 2010 Phoenix College workshop to learn more about copyright/fair use. However, you chose not to attend the workshop.

You are reminded that Section 4 of Administrative Regulation 3.2, quoted above, clearly advises that the Governing Board prohibits and disapproves of unauthorized duplication in any form. Section 5 of the Regulation further advises that employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

You are finally admonished that any further copyright/fair use violations will lead to disciplinary action being taken against you, up to and including termination of your employment.

I am willing to arrange for individual training and consulting on copyright if you feel it would be helpful. But I will restore your copying privileges only when I determine that you not only understand your legal obligations, but also that you will strictly adhere to them.

Sincerely,



Anna Solley, Ed.D.
President

cc: Mr. Paul DeRose
Mr. Richard Galvan
Ms. Casandra Kakar
Ms. Maggie McConnell

EEOC000192

MCCCD/Martinez 01369

Exhibit D

EXPERT REPORT OF
SEAN D. GARRISON

In the Matter of:

*Martinez v. Maricopa County Community
College District and Phoenix College*

Case No. 2:12-cv-00702-DCG

APRIL 19, 2013

GARRISON 0001

MCCCD/Martinez 01371

I have been retained by Maricopa County Community College District (the "District") in the matter of *Martinez v. Maricopa County Community College District and Phoenix College*, Case No. 2:12-cv-00702-DGC, to provide an expert opinion and report based upon my analysis of 4 documents, which I understand to be course packets and/or lecture notes created by Dr. Cleopatria Martinez intended for distribution to students in her math classes at Phoenix College. I have been asked to opine as to whether the documents are the independent creation of Dr. Martinez and, if not, whether the ongoing use of such documents by Dr. Martinez creates any risk that the District could be subject to a claim of copyright infringement.

A summary of my opinions in this matter follows immediately below.

SUMMARY OF OPINIONS

- The document entitled "MAT 182 Precalculus Trigonometry Section Lecture Notes Fall 2009" (Exhibit 1) is not the original creation of Dr. Martinez and, if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, would subject the District to a serious risk of a copyright infringement claim
- The document entitled "MAT 182 Trigonometry Spring 2010" (Exhibit 2) is not the original creation of Dr. Martinez and, if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, would subject the District to a serious risk of a copyright infringement claim
- The document entitled "MAT 082 Chapters for Basic Arithmetic Spring 2010" (Exhibit 3) is not the original creation of Dr. Martinez and, if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, would subject the District to a serious risk of a copyright infringement claim
- There is just reason for concern that the document entitled "MAT 187 Precalculus Algebra Section Lecture Notes Fall 2010" (Exhibit 4) is not the original creation of Dr. Martinez and was copied, at least in part, from one or more other sources
- None of the Course Materials complies with the Classroom Guidelines articulated by the representatives of the Ad Hoc Committee of Educational Institutions and Organizations on Copyright Law Revision, and of the Authors League of America, Inc., and the Association of American Publishers, Inc., which were reproduced and incorporated in the House Report (H.R. Rep. No. 94-1476) concerning the 1976 revisions to the Copyright Act
- None of the Course Materials complies with the Permissions Documentation solicited by Dr. Martinez from Pearson Education, Inc. in the Fall of 2010
- If the District were forced to defend a copyright infringement claim with respect to any of the Course Materials, the defense costs alone would undoubtedly be in the hundreds of thousands of dollars if not more

- It was appropriate for the District to take proactive steps to prevent Dr. Martinez from using the Course Materials and subjecting the District to an undue risk of copyright infringement claims

FACTUAL BACKGROUND

In 2009-2010, Dr. Cleopatria Martinez prepared a series of course packets and lecture notes (the "Course Materials") for distribution to and use by students in her mathematics courses at Phoenix College.

In October of 2010, I was contacted by the District and asked to analyze whether the Course Materials appear to have been independently created or copied from other sources. I subsequently met with Mr. Joe Sueyoshi (Math Department Chair at Phoenix College), Casandra Kakar (Vice President of Academic Affairs at Phoenix College), and Margaret McConnell, an in-house attorney for the District, at Phoenix College to discuss background of the matter. Thereafter, Nate Edwards, an associate at Lewis and Roca working under my direction, and I reviewed and analyzed the materials as requested. We prepared a memo to the District summarizing our analysis, concluding that at least three of the four Course Materials were not Dr. Martinez's independent creations, expressing concerns about the fourth, and recommending that the District not permit further distribution and use of the materials. A copy of that memo is attached as Exhibit 5.

COPYRIGHT LAW BACKGROUND

A party may obtain copyright protection for "original works of authorship fixed in any tangible medium of expression." 17 U.S.C. § 102(a). Only works of authorship that are original are protected by copyright law. *Id.* "Original, as the term is used in copyright, means only that the work was independently created...and that it possesses at least some minimal degree of creativity." *Feist Publications, Inc. v. Rural Tel. Service Co., Inc.*, 499 U.S. 340, 356 (1991).

The elements of a copyright-infringement claim are (1) ownership of the copyright by the plaintiff and (2) copying by the defendant. *See Feist*, 499 U.S. at 361.

Facts, including mathematic definitions, formulas and calculations are not protectable by copyright. *See Educational Testing Services v. Katzman*, 793 F.2d 533, 536-37 (3d Cir. 1986); *see also Chicago Bd. of Educ. v. Substance, Inc.*, 354 F.3d 624 (7th Cir.2003) (holding copyright protection exists for exam questions). Although facts are not subject to copyright protection, a "compilation" of facts may be entitled to protection. *Katzman*, 793 F.2d at 536-37; 17 U.S.C. §103(a). But, that protection extends only to the original material added by the author of the compilation, not to facts contained within the compilation. In other words, a single mathematic equation or calculation is not entitled to copyright protection, but a compilation of equations and calculations may be.

Math word problems and math exam questions can be protected expression if they possess "some minimal degree of creativity." *See Feist Publications*, 499 U.S. at 356. Mathematic word problems are protectable where they do not represent the only means of expressing the equations/calculations on which they are based. *Educational Testing Service*, 793

F.2d at 540. In other words, the mere fact that word problems refer to mathematic equations or facts - or require use of equations or facts to solve them - does not, by itself, deprive the problems of copyright protection. *See id.* (finding that there are numerous means to frame questions to test students knowledge of “square roots or dangling participles” and that questions testing this knowledge were protected under copyright law). Therefore, because word problems can reflect original expression in their word choice and explanations, they can be protected by copyright even if they test knowledge of established mathematic formulas.

The Doctrine of Fair Use

Not all copying of protected expression necessarily subjects the copier to liability for copyright infringement. Initially adopted and developed by the courts, the doctrine of fair use was codified by Congress in the 1976 Copyright Act. Specifically, section 107 (17 U.S.C. §107) provides:

[T]he fair use of a copyrighted work, including such use by reproduction in copies ... for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright. In determining whether the use made of a work in any particular case is a fair use the factors to be considered shall include-

- (1) the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;
- (2) the nature of the copyrighted work;
- (3) the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and
- (4) the effect of the use upon the potential market for or value of the copyrighted work....

This language does not provide blanket immunity to teachers for “multiple copies for classroom use.” *Princeton Univ. Press v. Mich. Doc. Servs., Inc.*, 99 F.3d 1381, 1385 (6th Cir.1996) (en banc), *cert. denied*, 520 U.S. 1156, 117 S.Ct. 1336, 137 L.Ed.2d 495 (1997). Instead, “whether a use referred to in the first sentence of Section 107 is a fair use in a particular case ... depend[s] upon the application of the determinative factors.” *Campbell v. Acuff-Rose Music, Inc.*, 510 U.S. 569, 578, n. 9, 114 S.Ct. 1164, 1170 n.9, 127 L.Ed.2d 500 (1994), quoting S.Rep. No. 94-473, p. 62. No single factor is determinative, although factor (4) has emerged generally to be considered the most important. *Nimmer on Copyright*, Vol. 4, § 13.05[A][4], p. 13-198.2 (2012 ed.).

In evaluating the various fair use factors, the following principles should be observed:

With regard to factor (1), the purpose and character of the use, the Supreme Court has stated that the central purpose of the investigation of this factor is to determine whether “the new work merely supersedes the objects” of the original work, or whether it adds something new or

different in purpose or character; that is, is the new work “transformative”? *Campbell*, 510 U.S. at 579.

With regard to factor (2), the nature of the copyrighted work, the more creative the work, the greater protection it should receive against a finding of fair use; conversely, the more factual and informational a work, the broader the scope of finding fair use. *Leadsinger, Inc. v. BMG Music Publ'g*, 512 F.3d. 522, 531 (9th Cir. 2007).

With regard to factor (3), the amount and substantiality of the portion used, the inquiry is both quantitative and qualitative. *Nimmer on Copyright*, Vol. 4, § 13.05[A][3], p. 13-198. Copyright infringement can occur even if the amount of material infringed upon is low if the copied material is the core of the protected material. *Harper & Row Publishing, Inc. v. Nation Entertainment*, 471 U.S. 539, 565 (1985) (300 words of a full length book manuscript was not a fair use).

With regard to factor (4), the effect of the use upon the potential market for or value of the copyrighted work, this inquiry is not limited merely to the single defendant's conduct, but instead considers whether the widespread and unrestricted conduct of the sort engaged in by the defendant would adversely impact the potential market or value of the copyright owner's work. *Nimmer on Copyright*, Vol. 4, § 13.05[A][3], pp. 13-198.3 - 13-198.4; *American Geophysical Union v. Texaco, Inc.*, 60 F.3d 913, 927 (2d Cir. 1994) (finding no fair use for researchers to copy and distribute internally article from the plaintiff's publication).

Agreement on Guidelines for Classroom Copying in Not-For-Profit Educational Institutions
With Respect to Books and Periodicals

In connection with the revisions to the Copyright Act made in 1976, and the enactment of the new statute, an ad hoc committee consisting of representatives of the author-publisher and educational organization constituencies was formed to discuss the issues of fair use of copyrighted works in the classroom by not-for-profit educational institutions. The committee members agreed upon a set of guidelines that it believed should “state the minimum and not the maximum standards of educational fair use.” In other words, whether copying that exceeds the stated guidelines should be considered a fair use would be left to the courts.

In a joint letter to House Committee Chairman Kastenmeier, dated March 19, 1976, the representatives of the Ad Hoc Committee of Educational Institutions and Organizations on Copyright Law Revision, and of the Authors League of America, Inc., and the Association of American Publishers, Inc., communicated its agreed upon guidelines, which were reproduced and incorporated in the House Report (H.R. Rep. No. 94-1476).¹ The letter and guidelines are set out immediately below:

¹ Excerpts from the House Report are reproduced in Copyright Office Circular 21, “Reproduction of Copyrighted Works by Educators and Librarians,” which is attached to this Report as Exhibit 6. I will refer to these guidelines hereafter as the “Classroom Guidelines.”

The purpose of the following guidelines is to state the minimum and not the maximum standards of educational fair use under Section 107 of H.R. 2223. The parties agree that the conditions determining the extent of permissible copying for educational purposes may change in the future; that certain types of copying permitted under these guidelines may not be permissible in the future; and conversely that in the future other types of copying not permitted under these guidelines may be permissible under revised guidelines. Moreover, the following statement of guidelines is not intended to limit the types of copying permitted under the standards of fair use under judicial decision and which are stated in Section 107 of the Copyright Revision Bill. There may be instances in which copying which does not fall within the guidelines stated below may nonetheless be permitted under the criteria of fair use.

Guidelines

I. Single Copying for Teachers

A single copy may be made of any of the following by or for a teacher at his or her individual request for his or her scholarly research or use in teaching or preparation to teach a class:

- A. A chapter from a book
- B. An article from a periodical or newspaper
- C. A short story, short essay or short poem, whether or not from a collective work
- D. A chart, graph, diagram, drawing, cartoon or picture from a book, periodical, or newspaper

II. Multiple Copies for Classroom Use

Multiple copies (not to exceed in any event more than one copy per pupil in a course) may be made by or for the teacher giving the course for classroom use or discussion; provided that:

- A. The copying meets the tests of brevity and spontaneity as defined below and,
- B. Meets the cumulative effect test as defined below and,
- C. Each copy includes a notice of copyright

Definitions

Brevity

i Poetry: (a) A complete poem if less than 250 words and if printed on not more than two pages or, (b) from a longer poem, an excerpt of not more than 250 words.

ii Prose: (a) Either a complete article, story or essay of less than 2,500 words, or (b) an excerpt from any prose work of not more than 1,000 words or 10% of the work, whichever is less, but in any event a minimum of 500 words.

[Each of the numerical limits stated in “i” and “ii” above may be expanded to permit the completion of an unfinished line of a poem or of an unfinished prose paragraph.]

iii Illustration: One chart, graph, diagram, drawing, cartoon or picture per book or per periodical issue.

iv “Special” works: Certain works in poetry, prose or in “poetic prose” which often combine language with illustrations and which are intended sometimes for children and at other times for a more general audience fall short of 2,500 words in their entirety. Paragraph “ii” above notwithstanding such “special works” may not be reproduced in their entirety; however, an excerpt comprising not more than two of the published pages of such special work and containing not more than ten percent of the words found in the text thereof, may be reproduced.

Spontaneity

- i The copying is at the instance and inspiration of the individual teacher, and
- ii The inspiration and decision to use the work and the moment of its use for maximum teaching effectiveness are so close in time that it would be unreasonable to expect a timely reply to a request for permission.

Cumulative Effect

- i The copying of the material is for only one course in the school in which the copies are made.
- ii Not more than one short poem, article, story, essay or two excerpts may be copied from the same author, nor more than three from the same collective work or periodical volume during one class term.
- iii There shall not be more than nine instances of such multiple copying for one course during one class term.

[The limitations stated in “ii” and “iii” above shall not apply to current news periodicals and newspapers and current news sections of other periodicals.]

III. Prohibitions as to I and II Above

Notwithstanding any of the above, the following shall be prohibited:

- A. Copying shall not be used to create or to replace or substitute for anthologies, compilations or collective works. Such replacement or substitution may occur whether copies of various works or excerpts therefrom are accumulated or reproduced and used separately.
- B. There shall be no copying of or from works intended to be “consumable” in the course of study or of teaching. These include workbooks, exercises, standardized tests and test booklets and answer sheets and like consumable material.

C. Copying shall not:

- a) substitute for the purchase of books, publishers' reprints or periodicals;
- b) be directed by higher authority;
- c) be repeated with respect to the same item by the same teacher from term to term.

D. No charge shall be made to the student beyond the actual cost of the photocopying.

Agreed March 19, 1976.

Ad Hoc Committee on Copyright Law Revision:

By Sheldon Elliott Steinbach.

Author-Publisher Group:

Authors League of America:

By Irwin Karp, Counsel.

Association of American Publishers, Inc.:

By Alexander C. Hoffman,

Chairman, Copyright Committee.

Although the foregoing guidelines are not controlling on the court, they have been referred to and relied upon by courts in fair use cases involving copying for educational institutions. See *Marcus v. Rowley*, 695 F.2d 1171, 1178 (9th Cir. 1983) (a teacher's retyping of substantial portion of another teacher's copyrighted work held not be a fair use even though intended for use in the classroom); *Mich. Doc. Servs., Inc.*, 99 F.3d at 1391; *Basic Books, Inc. v. Kinko's Graphics Corp.*, 758 F. Supp. 1522, 1535 (S.D.N.Y. 1991).

The Rise of Printed Coursebooks and Related Copyright Infringement Cases

Kinko's Copies Corp. was founded in 1970 at location near University of California at Santa Barbara. "By the mid-1970s Kinko's was providing custom publishing materials for colleges, an innovation extremely popular with college professors. The company had 80 stores, averaging 400 square feet in space and located primarily near colleges and universities, by the end of the decade." *Kinko's Inc. History*, <http://www.fundinguniverse.com/company-histories/kinko-s-inc-history/> (accessed April 18, 2013)². As Kinko's and other print shops opened up around college campuses, they began copying and printing coursebooks for university professors.

As reported on The Copyright Site, <http://www.thecopyrightsite.org/scenarios/coursepacks.html>:

In 1982 several publishing companies brought a law suit against NYU, nine faculty members, and a photocopying establishment. The Association of American Publishers which coordinated the law suit on behalf of the publishing companies sought an injunction to keep the professors from copying course

² A printout of this article is attached as Exhibit 7 to the Report.

anthologies. The publishers reached an out-of-court settlement with the defendants thus requiring the university to appropriate, promulgate, and invoke more stringent copyright rules.³

The news of this settlement led to other publishers seeking to enforce their rights and cases being filed in the courts. At issue in these cases was whether the creation and sale of course packets to students was a fair use for educational purposes. One of the first decisions involved Kinko's. *Basic Books, Inc. v. Kinko's Graphics Corp.*, 758 F. Supp. 1522, 1535 (S.D.N.Y. 1991). The court found that Kinko's copying and preparation of course packets for students was not a fair use under the copyright statute. Similar cases followed. *See Princeton Univ. Press v. Mich. Doc. Servs., Inc.*, 99 F.3d 1381, 1385 (6th Cir.1996) (en banc), *cert. denied*, 520 U.S. 1156, 117 S.Ct. 1336, 137 L.Ed.2d 495 (1997); *Blackwell Publ'g Inc. v. Excel Research Group LLC*, 661 F. Supp.2d 786 (E.D. Mich. 2009).

THE FINANCIAL CONSEQUENCES OF A COPYRIGHT INFRINGEMENT CLAIM

Copyright Infringement Remedies and Criminal Penalties

The Copyright Act, 17 U.S.C. § 101 et seq., sets forth the civil remedies that are available to a copyright owner whose work has been infringed. These remedies may include injunctive relief, damages and the recovery of attorney's fees. 17 U.S.C. §§ 502-505.

With respect to damages, a copyright owner is always entitled to the actual damages it has suffered as a result of the infringement, plus any additional profits of the infringer that are attributable to the infringement and not taken into account in computing the copyright owner's actual damages. 17 U.S.C. § 504(b). Here, the textbook publishers would have two potential avenues to pursue damages against the District if Dr. Martinez's Course Materials were ultimately judged to infringe. First, if it were determined that students who were given the Course Materials did not purchase the textbook(s) used as the source of infringing content, then the publisher(s) of the textbook would have a claim for lost profits on the sale of the textbook(s). Second, the publisher(s) might make an infringer's claim for all or a portion of the course tuition paid by the students to the District with respect to the particular math class.

As an alternative to pursuing actual damages, the owner of a registered copyright may instead pursue statutory damages for the infringement.⁴ 17 U.S.C. § 504(c). Under this section, a copyright owner does not have to prove any actual damages from the infringement. Instead, the owner may ask the court to award statutory damages in an amount of \$750 to \$30,000 per work for non-willful infringement. If the infringement is determined to be willful, then the court may award up to \$150,000 per work. The copyright owner may elect this remedy at any time before a final judgment is entered. *Id.* Here, there are at least 3 textbooks at issue. If, upon becoming aware of a potential infringement issue, the District had failed to take steps to prevent

³ Copyright Scenarios, <http://www.thecopyrightsite.org/scenarios/coursepacks.html>, accessed April 18, 2013. A printout of this article is attached as Exhibit 8 to this Report.

⁴ I searched the U.S. Copyright Office records and confirmed that each of the textbooks referred to in this report is the subject of a registered copyright. Accordingly both statutory damages and attorney's fees would be available to the publishers if they successfully proved copyright infringement at trial.

copyright infringement, and a judge or jury were to determine that each of these textbooks were willfully infringed by Dr. Martínez, then the total statutory damages award could be as high as \$450,000.

Finally, a prevailing copyright owner is entitled to an award of its costs and reasonable attorney's fees incurred in litigating the matter. 17 U.S.C. § 505. In deciding whether to award fees, the district court should consider the degree of success obtained; frivolousness; motivation; objective unreasonableness (both in the factual and legal arguments in the case); and the need in particular circumstances to advance considerations of compensation and deterrence. *Fogerty v. Fantasy, Inc.*, 510 U.S. 517, 535 n. 19, 114 S.Ct. 1023, 127 L.Ed.2d 455 (1994) (listing factors to be considered for awarding attorney's fees in copyright actions). Any award of fees is within the trial court's discretion. *Id.* at 534.

As discussed in the section immediately following, the average cost of litigation a copyright case ranges from several hundred thousand to over a million dollars. *See also Unicom Systems, Inc. v. Farmers Group, Inc.*, 405 Fed. Appx. 152 (9th Cir. 2010) (affirming fee award of over \$1.5 million); *Range Road Music, Inc. v. East Coast Foods, Inc.*, 668 F.3d 1148 (9th Cir. 2012) (affirming fee award of \$162,728.22).

In addition, 17 U.S.C. § 506 provides that willful infringement of a copyright may in certain circumstances result in criminal penalties, including imprisonment and fines under 18 U.S.C. § 2319.

Copyright Infringement Litigation Expense

It is very expensive to defend copyright infringement claims. Just one case would likely cost the District hundreds of thousands, if not millions, of dollars to defend. The American Intellectual Property Law Association (AIPLA) publishes a bi-annual economic survey entitled, "Report of the Economic Survey," in which it reports on a variety of economic issues pertaining to the acquisition and enforcement of intellectual property rights. This Report includes information on the cost of intellectual property litigation. According to the most recent edition of the AIPLA Report, the average copyright case filed in Arizona, in which there is less than \$1 million in damages at issue, would cost a party \$344,000 to take through trial. When there is between \$1 million and \$25 million at stake, the average cost of a lawsuit rises to \$1,064,000.⁵

Accordingly, even in cases where the District has a legitimate defense to the claim, the economic consequences of being sued are immense. As discussed above, if the District did not prevail, and a judgment was entered against it, the consequences would be even more severe. Not only would the District be forced to pay its own attorneys and whatever damages might be awarded to the plaintiff, but it could also be ordered to pay the plaintiff's attorney's fees.

In light of both the substantial monetary awards and fines that could be imposed in a copyright infringement case, as well as the sheer cost to defend an infringement action, the potential financial consequences to the District in a copyright infringement case are huge.

⁵ AIPLA, *2011 Report of the Economic Survey*, pp. I-163 and I-164. Relevant excerpts from the AIPLA Report are attached as Exhibit 9 to this Report.

Accordingly, it becomes important for the District to act proactively, whenever reasonably possible, to avoid such claims.

RESULTS OF COURSE MATERIALS REVIEW

For purposes of this Report, I have reviewed the following entitled packets:

- MAT 182 Precalculus Trigonometry Section Lecture Notes Fall 2009 (Exhibit 1)
- MAT 182 Trigonometry Spring 2010 (Exhibit 2)
- MAT 082 Chapters for Basic Arithmetic Spring 2010 (Exhibit 3)
- MAT 187 Precalculus Algebra Section Lecture Notes Fall 2010 (Exhibit 4)

My review of the Course Materials indicates that at least significant portions of three of these Course Materials were copied directly from other sources, including *Basic Mathematics* by Charles P. McKeague (published by Cengage Education); *Prealgebra* by Alan R. Tussy and R. David Gustafson (published by Thomson Learning, Inc.); and *Precalculus* by Michael Sullivan and Michael Sullivan, III (published by Pearson Education, Inc.).

None of the Course Materials includes any attribution to these or any other source texts, which indicates that Dr. Martinez is portraying these works as her own independent creations. In the MAT 187 notes documents, Dr. Martinez does generically thank a list of individuals for their inspiration, ideas, comments and suggestions. However, none of these is the publisher of any textbooks.

Documentation of Permission to Use Problems and Examples from Textbooks

I have reviewed several documents consisting of what I understand to be communications between Dr. Martinez and representatives Pearson Education, Inc. - the publisher of the Sullivan textbook - concerning her request to use content from the Sullivan textbook in her course materials and tests. Specifically, these documents are identified as Bates Numbers EEOC 000136 - EEOC 00143. These communications occurred between April and October of 2010 and will be referred to collectively in this report as the "Permissions Documentation":

- April 13, 2010 Fax (EEOC 000136-000137)
- May 13, 2010 Fax (EEOC 000138-000139)
- August 23, 2010 email from Dr. Martinez to Estelle Simpson (EEOC 000140)
- October 8, 2010 email from Estelle Simpson to Dr. Martinez (EEOC 000141)
- Series of October 2010 emails between Dr. Martinez and Ms. Simpson (EEOC 000142-000143)

(1) The April 13, 2010 Fax

The April 13, 2010 fax provides permission from Pearson for Dr. Martinez to make up to 35 copies of selected Homework problems from Chapters 5-7 (pp. 371-563) of the work *Precalculus: Concepts Through Functions, A Right Triangle Approach* (2007 ed.) for the class MAT187 Precalculus beginning in the Fall of 2010 at Phoenix College. This permission is granted free of charge on the understanding that the textbook would be used and purchased by

students in the class. The permission is also contingent upon Dr. Martinez using the following acknowledgment on the authorized copies:

"Sullivan, PRECALCULUS: CONCEPTS THROUGH FUNCTIONS, A RIGHT TRIANGLE APPR, © 2007. Reprinted by permission of Pearson Education, Inc."

(2) The May 13, 2010 Fax

The May 13, 2010 fax provides permission from Pearson for Dr. Martinez to make up to 50 copies of selected Homework problems from Chapters 6-9 of the *Precalculus: Concepts Through Functions, A Right Triangle Approach* (2011 ed.) for the class MAT187 Precalculus beginning in the Fall of 2010 at Phoenix College. This permission is granted free of charge on the understanding that the textbook would be used and purchased by students in the class. The permission is also contingent upon Dr. Martinez using the following acknowledgment on the authorized copies:

"Sullivan, PRECALCULUS: CONCEPTS THROUGH FUNCTIONS, A RIGHT TRIANGLE APPR, © 2011. Reprinted by permission of Pearson Education, Inc."

(3) The August 23, 2010 email

The August 23, 2010 email correspondence addresses whether Dr. Martinez needs permission from Pearson to create and use in her lecture notes her own examples and problems that are "similar to the selected examples" contained in the Sullivan textbook. The Pearson representative advises that she does not need such permission. This makes perfect sense because, as described in the e-mail, Dr. Martinez would be creating her own examples, i.e. an independent creation.

(4) The October 8, 2010 email

The October 8, 2010 email correspondence addresses whether Dr. Martinez may incorporate actual examples from the Sullivan textbook in her class lecture notes distributed to the students. The Pearson representative advises that she may do so only if the textbook is being used for the class and purchased by the students in the class. This e-mail exchange can be viewed as a follow-on to the April 13 and May 13 faxes discussed above.

(5) The series of additional October 2010 emails

The series of emails from October 13 to October 19, 2010 concerns whether Dr. Martinez may use problems from two different textbooks - the Sullivan *Precalculus* textbook and the Briggs/Cochran *Calculus* textbook - in the tests she gives to her students. Again, the Pearson representative advises that she may do so only if the textbooks are being used for the class and purchased by the students in the class.

These emails apply only to exams and tests and not to any course packets or lecture notes. Accordingly, they are not pertinent to my analysis here, except as evidence that the textbook publishers are diligent and specific with respect to the manner in which they authorize use of excerpts of textbooks for in class usage.

The Permissions Documentation is attached collectively as Exhibit 10 to this Report.

Analysis of Course Materials

Presented below are charts detailing discrepancies in the text of the Course Materials and instances where I discovered content of the Course Materials to be substantially similar or identical to the content in other textbook sources.

Precalculus Trigonometry Section Fall 2009 (MAT 182)

Although the cover page states "Fall 2009," the heading on the internal pages shows Spring 2010, which suggests that these lecture notes were used in consecutive semesters. These lecture notes were intentionally and systematically copied from the Sullivan book *Precalculus, Concepts Through Functions*, Instructor's Edition. It is my understanding, however, that this textbook was not used in the Spring 2010 MAT182 class. Indeed, Dr. Martinez has acknowledged in writing that one of the reasons she created these kinds of materials for her students was because "many of my students cannot afford to buy a math textbook."⁶

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
Page 6 of 73 Section 1.2	Problems in Section 1.2 copied directly from source text	<i>Precalculus</i> , Sullivan, p. 371-372
Page 8 of 73	Example for Area of a Sector is copied from source text	<i>Precalculus</i> , Sullivan, p. 369 (Example 7)
Page 10 of 73 Section 1.3	Arc length problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 372
Page 14 of 73 Section 1.4	Right Triangle homework problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 383-384
Page 16 of 73	Example 6 copied from source text; no examples 1-5 appear in this section of the course materials	<i>Precalculus</i> , Sullivan, p. 382
Page 17 of 73 Section 1.5	Pythagorean Identity problems are copied from source text	<i>Precalculus</i> , Sullivan, p. 384
Page 20 of 73 - Page 21 of 73	Examples 1 -5 are identical to source text	<i>Precalculus</i> , Sullivan, p. 387-390
Page 22 of 73 Section 1.6	Problems (including introductory text) copied from source text; "Projectile Motion" section (including the image) is copied verbatim from source text	<i>Precalculus</i> , Sullivan, p. 393-394
Page 23 of 73	Examples 1 and 2 copied substantially from source text; includes some verbatim copying	<i>Precalculus</i> , Sullivan, p. 398-399
Page 24 of 73	Examples 3 and 4 copied substantially from source text; includes some verbatim copying.	<i>Precalculus</i> , Sullivan, p. 400-402
Page 25 of 73	Examples 5-7 copied substantially from source text; includes some verbatim copying	<i>Precalculus</i> , Sullivan, p. 402-404

⁶ A copy of Dr. Martinez's email is attached as Exhibit 11.

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
Page 26 of 73 Section 1.7	Problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 406-407
Page 29 of 73 Section 1.8	Circle Trig homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 416
Page 34 of 73 Section 1.9	Graphs & Transformations problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 429-430
Page 35 of 73	Tables are copied from Table 9 and Figure 93 of source text	<i>Precalculus</i> , Sullivan, p. 433
Page 37 of 73 Section 1.10	Graphing the Tangent problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 437-438
Page 44 of 73 Section 1.11	Inverse Sine problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 469-470
Page 47 of 73 Section 1.12	Values of Inverse Trig Functions Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 476-477
Page 51 of 73 Section 1.13	Trigonometric Identities Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 483
Page 57 of 73 - Page 58 of 73 Section 1.14	Sum and Difference Formulas Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 492-493
Page 61 of 73 - Page 62 of 73 Section 1.15	Double-angle problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 501-502
Page 65 of 73 - Page 66 of 73 Section 1.16	Solving Trig Equations problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 510-511 and 518-519
Page 67 Section 1.17	Applications Involving Right Triangle problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 531
Page 71 of 73 - Page 72 of 73 Section 1.18	Law of Sines homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 539-540
Page 73 of 73 - Page 74 of 74 Section 1.19	Law of Cosines homework problems copied directly from source text.	<i>Precalculus</i> , Sullivan, p. 547

First, in light of the substantial copying evidence above, I conclude that these lecture notes are not the original creation of Dr. Martinez. At best, they constitute a derivative work that would likely require permission from the copyright owner of the original work (Pearson Education, Inc.)

For several reasons, none of the Permissions Documentation appears to be applicable to this document. First, the Permissions Documentation states that the permission requested and authorized is to begin in the Fall of 2010. These lecture notes state Fall of 2009 and Spring of 2010, which pre-dates the effective permission date. Second, the Permissions Documentation (EEOC 000136-000139) applies only to the MAT187 - Precalculus class, not MAT182. Third, these lecture notes do not contain the acknowledgment required by the Permissions Documentation. Finally, I understand from Mr. Joseph Sueyoshi (Phoenix College Math Department Chair) that Dr. Martinez did not use the Sullivan *Precalculus* textbook in her Spring 2010 MAT182 class. I have not been provided with any other documentation indicating that Dr. Martinez received permission to copy from the Sullivan *Precalculus* textbook for her Fall 2009

or Spring 2010 MAT182 class. Accordingly, I conclude that if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, her use and distribution of them would subject the District to a serious risk of a copyright infringement claim by Pearson Education, Inc.

With respect to the four statutory fair use factors, these lecture notes do not appear to be transformative in any significant way; they merely copy examples from the textbook. The notes have the potential to serve as a replacement for the textbook in the event that students were not required to purchase the textbook for the class. An arguably substantial portion, both quantitative and qualitative, of the textbook has been copied. All three of these factors weigh against a finding of fair use. The only factor that clearly falls in favor of fair use is the nature of the copyrighted work itself.

In addition, these excerpts do not meet the "Classroom Guidelines" for fair use in the House Report. First, the copying does not meet the brevity, spontaneity or cumulative effect limitations in the Classroom Guidelines. Second, the lecture notes do not contain any copyright notices as mandated in Section IIC of the Classroom Guidelines. To the extent the textbook was not used and purchased by students in the class, these lecture notes would not comply with Prohibition IIC(a). To the extent these notes are intended to be re-used in subsequent semesters, they would not comply with Prohibition IIC(c).

Because I have been provided no documentation indicating that this copying has been authorized by Pearson Education, Inc., I conclude that if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, her use and distribution of them would subject the District to a serious risk of a copyright infringement claim by Pearson Education, Inc.

MAT 182 Trigonometry Spring 2010

This second set of materials reviewed consist of 50 pages with the heading MAT182 Trigonometry Spring 2010. The first 24 pages of this document are numbered "Page 1 of 24" to "Page 24 of 24"; the 25th page is numbered "Page 25 of 50," and that pagination header continues to the end of the document (Page 50 of 50).

These materials do not contain homework problems but appear to be handouts with examples to be used for taking class notes. I understand this document was obtained from a student in Dr. Martinez's Spring 2010 MAT 182 class. It includes what appear to be handwritten notes on various pages of the document. A review of these materials show that the incorporated examples were also copied from the Sullivan *Precalculus* textbook.

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
Page 1 of 24	Trig Functions of General Angles taken from the source text	<i>Precalculus</i> , Sullivan, p. 398 (Examples 1 and 2)
Page 5 of 24	Domain & Range Trig Functions	<i>Precalculus</i> , Sullivan, p. 413-414 (Example 2)
Page 6 of 24	Using even and odd properties	<i>Precalculus</i> , Sullivan, p. 414-415 (Example 3)

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
Page 9 of 24	Examples for graphing sine and cosine identical to source text	<i>Precalculus</i> , Sullivan, p. 419 (Example 1) and 421 (Example 3)
Page 10 of 24	$y=2\cos x$ text and graph tracks source text	<i>Precalculus</i> , Sullivan, p. 422-423
Page 11 of 24	Examples for amplitude identical to source text	<i>Precalculus</i> , Sullivan, p. 425 (Example 6) and p. 426 (Example 7)
Page 12 of 24	Substantially copied from Table 9 and Figure 93 in source text	<i>Precalculus</i> , Sullivan, p. 433
Page 13 of 24	Bottom graphs ($y = \cot x$) appears to be a direct copy from the text	<i>Precalculus</i> , Sullivan, p. 436
Page 14 of 24	Top graph ($y = \csc x$) appears to be a direct copy from the text	<i>Precalculus</i> , Sullivan, p. 436
Page 15 of 24 - Page 17 of 24	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 461-469 (Examples 1-9)
Page 21 of 24 - Page 22 of 24	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 472-475 (Examples 1, 2, 4, 6)
Page 25 of 50 - Page 26 of 50	Copies Trig Identities examples from textbook and merely changes the order of appearance.	<i>Precalculus</i> , Sullivan, p. 479 (Example 1)
Page 27 of 50 - Page 28 of 50	Copies "More examples" directly from textbook; summarizes "Guidelines for Establishing Identities" from box in textbook	<i>Precalculus</i> , Sullivan, p. 479-482 (Examples 2-8)
Page 31 of 50 - Page 32 of 50	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 488-490 (Examples 4-8)
Page 39 of 50 - Page 40 of 50	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 507-508 (Examples 1-4) and 514 (Example 1)
Page 41 of 50	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 414-515 (Examples 2-4)
Page 42 of 50	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 528 (Examples 1 and 2)
Page 43 of 50	Word problem is an identical copy to Example 3 in textbook	<i>Precalculus</i> , Sullivan, p. 529 (Example 3)
Page 44 of 50	Law of Sines text and images are identical to textbook	<i>Precalculus</i> , Sullivan, p. 533
Page 45 of 50	"Solve the oblique triangle" text is virtually verbatim	<i>Precalculus</i> , Sullivan, p. 533
Page 46 of 50 - Page 48 of 50	Law of Sines problem examples copied directly from textbook	<i>Precalculus</i> , Sullivan, p. 534-537 (Examples 1-5)
Page 49 of 50 - Page 50 of 50	Law of Cosines problem examples are copied directly from textbook	<i>Precalculus</i> , Sullivan, p. 545 (Examples 1 and 2)

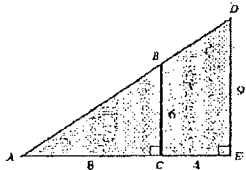
For the very same reasons as stated above with respect to the "Precalculus Trigonometry Section Fall 2009 (MAT 182)" lecture notes, the Permissions Documentation sought by Dr. Martinez do not appear to be applicable to these lecture notes.

With respect to the four statutory fair use factors, these lecture notes do not appear to be transformative in any significant way; they merely copy examples from the textbook. The notes have the potential to serve as a replacement for the textbook in the event that students were not required to purchase the textbook for the class. An arguably substantial portion, both quantitative and qualitative, of the textbook has been copied. All three of these factors weigh against a finding of fair use. The only factor that clearly falls in favor of fair use is the nature of the copyrighted work itself.

In addition, these excerpts do not meet the "Classroom Guidelines" for fair use in the House Report. First, the copying does not meet the brevity, spontaneity or cumulative effect limitations in the Classroom Guidelines. Second, the lecture notes do not contain any copyright notices as mandated in Section IIC of the Classroom Guidelines. To the extent the textbook is not used and purchased by students in the class, these lecture notes would not comply with Prohibition IIIC(a). To the extent these notes are intended to be re-used in subsequent classes, they would not comply with Prohibition IIIC(c).

Although basic formulas and/or symbols do not contain sufficient creativity to be copyrightable, the copying here is enough to create a significant risk that the compilation of the various formulae and examples used have been copied directly from the Sullivan textbook and are not the original work of Dr. Martinez. Because I have been provided no documentation indicating that this copying has been authorized by Pearson Education, Inc., I conclude that if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, her use and distribution of them would subject the District to a serious risk of a copyright infringement claim by Pearson Education, Inc.

Basic Arithmetic (MAT082)

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
Page 29 of 105; Page 30 of 105	Discrepancy: For the median and mode examples, the term "Juanita's Card Shop" is used. But, for the mean example, "Sarah's Card Shop" is used. In light of the evidence of other substantial copying, this is suggestive of copying a series of problems related to "Sarah's Card Shop" and changing "Sarah" to "Juanita" in some of the problems	N/A
Page 49 of 105, Nos. 1-24	Copies problems verbatim from textbook	<i>Basic Mathematics</i> ⁷ Charles P. McKeague p. 283
Page 49 of 105, Nos. 27 and 28	<p>Problem No. 27, corresponds with Chapter 4.1, Problem No. 36</p> <p>Identical image:</p>  <p>Identical text:</p> <ul style="list-style-type: none"> • "the diagram below, AC represents the length of the line segment that starts at A and ends at C" • "we see that $AC = 8$" • "a. Find the ratio of BC to AC" • "b. What is the length AE?" • "c. Find the ratio of DE to AE." <p>Problem No. 28, corresponds with Chapter 4.1, Problem No. 33</p> <p>These questions are nearly identical, the Course Materials replace "rent" with "house"</p>	<i>Basic Mathematics</i> Charles P. McKeague p. 285

⁷ These references were found through Internet searches, and screenshots from pages of the textbook are attached hereto as Exhibit 12.

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
	payment", and change the dollar amount of the house payment/rent. The Course Materials use a different graph. The problems are approximately 80% identical, the problem in the Course Materials appears immediately after the problem above and both problems appear in the same section of the outside source. Thus, we conclude there is a strong likelihood this problem was copied.	
Page 50 of 105, Nos. 29-32	<p><u>Problem No. 29 corresponds with Chapter 4.2, Problem No. 1</u> Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text. "A car travels 220 miles in 4 hours. What's the rate of the car in miles per hour?"</p> <p><u>Problem No. 30 corresponds with Chapter 4.2, Problem No. 2</u> Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text. "A train travels 360 miles in 5 hours. What's the rate of the train in miles per hour?"</p> <p><u>Problem No. 31 corresponds with Chapter 4.2, Problem No. 5</u> Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text. "The flow of water from a water faucet can fill a 3-gallon container in 15 seconds. Give the ratio of gallons to seconds as a rate in gallons per second."</p> <p><u>Problem No. 32 corresponds with Chapter 4.2, Problem No. 9</u> Identical text "A car travels 95 miles on 5 gallons of gas. Give the ratio of miles to gallons as a rate in miles per gallon."</p>	<i>Basic Mathematics</i> Charles P. McKeague p. 289
Page 60 of 105, Nos. 5 and 6	<p><u>Problem No. 5 corresponds with Chapter 5, Problem No. 66</u> Identical text except for omission of "by a stake" in Course Materials "A wire from the top of a 24-foot pole is fastened to the ground by a stake 10 feet from the bottom of the pole. How long is the wire?"</p> <p><u>Problem No. 6 corresponds with Chapter 5, Problem No. 65</u> Identical text "A ladder is leaning against the top of a 15-foot wall. If the bottom of the ladder is 20 feet from the wall, how long is the ladder?"</p>	<i>Prealgebra</i> Alan R. Tussy R. David Gustafson p. 420
Page 74 of 105, Nos. 1, 11 and 14-16	<p><u>Problem No. 1 corresponds with Chapter 9, Figure 9-8</u> Identical text "How many cubic feet of water are needed to fill a spherical water tank with a radius of 15 feet"</p> <p><u>Problem No. 11 corresponds with Chapter 9, Problem No. 50</u> Identical text "A classroom is 40 feet long, 30 feet wide, and 9 feet high. Find the number of cubic feet of air in the room."</p> <p><u>Problem No. 14 corresponds with Chapter 9, Problem No. 52</u> Identical text except Course Materials replace "advertised in a J.C. Penny catalog" with "in New Mexico" "The largest refrigerator advertised in a J.C. Penny catalog has a capacity of 25.2 cubic feet. How many cubic inches is this?"</p> <p><u>Problem No. 15 corresponds with Chapter 9, Problem No. 55</u> Identical text "The lifting power of a spherical balloon depends on its volume. How many cubic feet of gas will a balloon hold if it is 40 feet in diameter?"</p> <p><u>Problem No. 16 corresponds with Chapter 9, Problem No. 56</u> Identical text "A box of cereal measures 3 by 8 by 10 inches. The manufacturer plans to market a smaller</p>	<i>Prealgebra</i> Alan R. Tussy R. David Gustafson p. 615 (No. 1) p. 620 (Nos. 11, 14-16)

COURSE MATERIALS	DISCREPANCY OR TEXT	SOURCE INFORMATION
PAGE/QUESTION #		
	box that measures 2.5 by 7 by 8 inches. By how much will the volume be reduced?"	

For the very same reasons as stated above with respect to the "Precalculus Trigonometry Section Fall 2009 (MAT 182)" and "MAT182 Trigonometry Spring 2010" lecture notes, the Permissions Documentation sought by Dr. Martinez do not appear to be applicable to this set of course materials.

With respect to the four statutory fair use factors, these lecture notes do not appear to be transformative in any significant way; they merely copy examples from the textbook. The notes have the potential to serve as a replacement for one or both textbooks in the event that students were not required to purchase both textbooks for the class. An arguably substantial portion, both quantitative and qualitative, of the textbook has been copied. All three of these factors weigh against a finding of fair use. The only factor that clearly falls in favor of fair use is the nature of the copyrighted work itself.

In addition, these excerpts do not meet the "Classroom Guidelines" for fair use in the House Report. First, the copying does not meet the brevity, spontaneity or cumulative effect limitations in the Classroom Guidelines. Second, the lecture notes do not contain any copyright notices as mandated in Section IIC of the Classroom Guidelines. To the extent the textbooks are not used and purchased by students in the class, these lecture notes would not comply with Prohibition IIC(a). To the extent these notes are intended to be re-used in subsequent classes, they would not comply with Prohibition IIC(c).

In light of the substantial amount of verbatim copying contained in this set of materials, I conclude that this document is not the original work of Dr. Martinez. Because I have been provided no documentation indicating that this copying was authorized by the publishers (Cengage Learning and Thomson Learning, Inc.), I likewise conclude that if the District had not taken steps to prevent the copying and use of these materials by Dr. Martinez, her use and distribution of them would subject the District to a serious risk of a copyright infringement claim by Cengage Learning and/or Thomson Learning, Inc.

Precalculus Algebra Section Lecture Notes Fall 2010 (MAT 187)

This document was provided in Adobe PDF format that appears to have been created from a Word document. Although the cover pages indicates that these materials were created for use in the "Fall 2010" MAT187 course, the internal pages indicate "Spring 2010" in the header. This suggests that the content may have been utilized prior to the Fall of 2010. Due to the lack of any word problems in this work, and our limited access to potential source works, I have not determined whether this particular work is a work of independent creation by Dr. Martinez, whether she copied the problems from the Sullivan *Precalculus* textbook pursuant to the Permissions Documentation, or if she copied problems and equations from other works as is clearly the case in the other materials. I do note some irregularities within this document that are suspicious and suggest that at least some of the content in these lecture notes were copied from another source:

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
p. 21	In Homework for Symmetry, the introductory text states "In problems 29-37..." but the problems immediately following the intro paragraph are numbered 1-5, which suggests that these problems were copied from another source and simply renumbered.	N/A
p. 22	Graphs in problems 6-28 are not presented in a consistent appearance	N/A

I cannot state definitively at this time whether this particular work was created independently, nor is there sufficient evidence in the work itself for me to form an opinion that it was copied from other sources without permission. I understand that Dr. Martinez contends that she independently re-wrote these lecture notes over the summer of 2010. However, in light of the discrepancy found on page 21 and the history of copying evident in the other Course Materials, there is still reason for the District to be concerned.

Moreover, to the extent Dr. Martinez copied any of this content from the Sullivan *Precalculus* textbook, these lecture notes do not comply with the Permissions Documentation. Specifically, this document does not contain the required acknowledgment:

"Sullivan, PRECALCULUS: CONCEPTS THROUGH FUNCTIONS, A RIGHT TRIANGLE APPR, © 2011. Reprinted by permission of Pearson Education, Inc."

It is unknown whether any content may have copied from Chapters not covered by the Permissions Documentation or how many copies of these lecture notes were made. In any event, if these lecture notes were indeed used and distributed in Dr. Martinez's MAT187 Fall 2010 class, and were copied from the Sullivan *Precalculus* textbook, her failure to include the required acknowledgment would subject the District to a potential copyright infringement claim by Pearson Education, Inc. based on these lecture notes.

Finally, to the extent Dr. Martinez copied any of this content from the Sullivan *Precalculus* textbook or other textbooks, these excerpts do not meet the "Classroom Guidelines" for fair use in the House Report. First, the copying does not meet the brevity, spontaneity or cumulative effect limitations in the Classroom Guidelines. Second, the lecture notes do not contain any copyright notices as mandated in Section IIC of the Classroom Guidelines. To the extent the textbook is not used and purchased by students in the class, these lecture notes would not comply with Prohibition IIIC(a). To the extent these notes are intended to be re-used in subsequent classes, they would not comply with Prohibition IIIC(c).

CONCLUSION

For all the foregoing reasons, it was necessary and appropriate that the District took proactive measures, including imposing restrictions on copying, to prevent one of its professors from subjecting the District to a claim of copyright infringement by one or more textbook publishers.

STATEMENT OF COMPENSATION

With regard to my work in connection with the preparation of this Report and any testimony to be given in this case, I am charging the District \$500 per hour, plus reimbursement of out of pocket expenses.

A handwritten signature in black ink, appearing to be 'SB' followed by a long horizontal stroke.

April 19, 2013

Exhibit E



DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

VIA HANDDELIVERY

December 9, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

During the spring semester of 2010, I initiated an administrative evaluation based on Mr. Joe Sueyoshi's complaint, pursuant to section 3.7 of the Residential Faculty Policies Manual. By agreement, the Evaluation Team did not undertake its task until this fall semester.

I have reviewed the report of the Evaluation Team that was sent to me on November 17, 2010. The Evaluation Team which was advised by expert counsel, found that your work violated copyright. In addition, the Evaluation Team found that you were insubordinate and failed to follow District Legal Counsel's and my instructions and continued to request and persist in printing or copying unauthorized materials. The Evaluation Team agreed "that the administration should take appropriate disciplinary action following district policies in regards to this matter."

I have also considered our experiences under the current temporary arrangement in which Mr. Sueyoshi reviews materials you submit and may approve copying if they appear to be free of copyright and attribution problems. Given the Evaluation Team's findings and my lack of confidence in your willingness to follow the law as well as Maricopa policies and procedures due to your repeated copyright violations, I no longer consider the arrangement to be sufficient to protect the institution's legal and financial interests.

Pursuant to section 3.7.4 of the RFP, I now direct you to use only course materials approved by the department, that are available in the bookstore for sale to the students and that are authored by persons other than yourself. With department chair and Vice President for Academic Affairs approval, you may customize these materials for your students using traditional course packs or new online programs such as McGraw-Hill's "Create" (<http://create.mcgraw-hill.com/createonline/index.html#>) or McMillan's "Dynamic Books" (<http://dynamicbooks.vitalbook.com/>). All materials you use must be clearly attributed to authors other than yourself, and you must produce evidence of purchase or the author's permission to use them (e.g., a copy of the letter giving you a courtesy copy of the instructor's edition). If you wish to make copies of or from these materials, you must first produce written permission from the author and/or publisher to Mr. Sueyoshi. In addition, requests for the copying of these materials or your syllabi and quizzes/tests must be submitted in a timely manner (minimum of two work days notice) to Mr. Sueyoshi for approval.

MAIN CAMPUS
1202 West Thomas Road
Phoenix, AZ 85013
(602) 285-7433

PC DOWNTOWN CAMPUS
640 North 1st Avenue
Phoenix, AZ 85003
(602) 223-4000

GO FAR, CLOSE TO HOME.
phoenixcollege.edu

A Maricopa Community College.

MCCCD/Martinez01055

MCCCD/Martinez 01393

Dr. Cleopatria Martinez
December 9, 2010
Page 2

This direction is intended to communicate job duties to you within the meaning of Governing Board employment standard A4.3. In accordance with that section of the All Employee Policy Manual, willful and intentional violation of these instructions will be considered grounds for disciplinary action, up to and including termination of your employment.

Sincerely,

A handwritten signature in cursive script, appearing to read "Anna Solley", with a long horizontal flourish extending to the right.

Anna Solley, Ed.D.
President

cc: Mr. Lee Combs
Ms. Casandra Kakar
Mr. Joe Sueyoshi

EXHIBIT 16

3/1/2010

Copyright at MCCC

Hazel Davis, Faculty, Rio Salado College
Maggie McConnell, Assistant General Counsel
March 1st, 2010



The Faculty Perspective

Hazel Davis, Rio Salado College



Objectives

- Review the basics of copyright law
- Apply copyright law to the educational environment, including the Public Domain and Fair Use
- Explore the copyright issues affecting classroom copying and off-air taping
- Develop familiarity with free resources that can be used in teaching
- Determine how to request permissions




3/1/2010

What can be copyrighted?


- Four Requirements:
 - Expression of ideas
 - Original work of authorship
 - Modicum of creativity
 - “fixed in any tangible medium of expression, now known or later developed, from which [the works] can be perceived, reproduced or otherwise communicated, whether directly or with the aid of a machine or device”

© Copyright Law, Title 17, U.S. Code, Section 102




What Cannot be Copyrighted?


- Facts
- Ideas
- Short Phrases
 - (may be Trademarked)
- Titles
- Slogans
- Names
- Stock scenes
- Not written, recorded or captured electronically
- Laws from all government levels.
- U.S. Federal government documents



Copyright Owner's Rights



- the right to reproduce or copy
- the right to prepare derivative works
- the right to distribute copies to the public
- the right to perform or display the work publicly (for audiovisual, literary, dramatic, pictorial, etc.)



3/1/2010

Public Domain

- Anything Published before 1923
- Works produced by the U.S. federal government
 - Projects of non-government officials with federal funding may be copyrighted
 - State government works may be copyrighted



Public Domain Chart

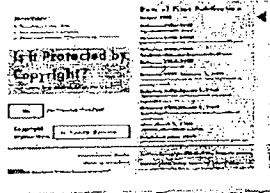
Adapted from "The Illustrated Story of Copyright," by Edward Samuel, p208.

Works created as of these dates	Duration of copyright	Explanation
Published before 1923	Already in Public Domain	Originally 56 years at most, extended to 75 years in 1976
Published 1923-1977	If published with © notice and renewed in 28 th year, copyright lasts for 55 years from publication date	Originally 28 years, renewable for 28 more years, extended to 75 years in 1976, extended to 95 years in 1998
Created after January 1, 1978	Life of author plus 70 years	Originally life plus 50, extended to life plus 70 in 1998 (Sonny Bono Extension)
Created before 1978, not yet published	Life plus 70 years or until 2002, whichever is later. If published by 2002, extended to 2047	As unpublished works, these would have gone on forever, but were given the fixed terms by the 1976 and 1998 acts



Copyright Slider

- An interactive Public Domain tool:
- <http://www.librarycopyright.net/digitalslider/>



3/1/2010

**Fair Use exemption—U.S. Code Title 17,
Section 107**

Four Factors

- Purpose and character of the use
 - How are you using the work?
- Nature of the work
 - What kind of work is it?
- Amount and substantiality of the portion
 - How much of the work are you using?
- Effect of the use
 - What effect will your use have on potential sales or permissions?



Factor I: Purpose of the Use

<u>Favoring Fair Use</u>	<u>Opposing Fair Use</u>
Teaching	Commercial
Research	Entertainment
Non-profit educational institution	For-profit educational institution
Personal	Failure to attribute to original author
Transformative Use— are you creating something new, or just copying?	



Factor II: Nature of the Work

<u>Favoring Fair Use</u>	<u>Opposing Fair Use</u>
Published work	Unpublished
Factual, non-fiction	Fiction
	Highly creative—art, film, music, etc.



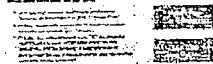
3/1/2010

Factor III: Amount of Work

<u>Favoring Fair Use</u>	<u>Opposing Fair Use</u>
Small amount	Large amount or whole work
Portion is not central to the work	Segment is central to the work, even if small amount—"heart of the work"

**Factor IV: Effect on Market**

<u>Favoring Fair Use</u>	<u>Opposing Fair Use</u>
Already own a copy	Replace sale of the work or affect market for permissions
Reproducing only 1 or a few copies	Making numerous copies
Restricted access	Making available on free web
No well-developed permissions market	Permission for use or licensing is readily available
	Long-term or repeat use

**Fair Use Checklists****Fair Use Evaluator**
<http://www.libraryofcongress.gov/fairuse>


3/1/2010

Classroom Guidelines

- Fair Use became part of Copyright Law in re-write of 1976
 - Guidelines For Classroom Copying in Not-For-Profit Educational Institutions emerged from discussions of 1976 law
- <http://www.unc.edu/~uncclng/classroom-guidelines.htm>



Classroom Guidelines

- Multiple copies
 - 1 copy per pupil per course
 - Needs to meet tests of brevity, spontaneity, and cumulative effect
 - Include copyright notice on each copy



Classroom Guidelines

- Brevity
 - 1 article, story or essay of less than 2,500 words
 - An excerpt from prose of less than 1,000 words or 10% of the work, whichever is less
 - One illustration per book or per article



3/1/2010

Classroom Guidelines

—Spontaneity

- Copying at the instance and inspiration of the individual teacher (no "higher authority")
 - The inspiration and decision to use the work make it unreasonable to expect a timely reply to a permission request.
- Intended for preserving the "teachable moment"



Classroom Guidelines

—Cumulative Effect

- Copies are for one course
- Not more than 1 short item or 2 excerpts from same author, not more than 3 from same work or periodical volume in 1 class term
- Not more than 9 instances of multiple copying for 1 course in 1 term



Classroom Guidelines Prohibitions

—Copying Shall Not:

- Substitute for the purchase of books, reprints or periodicals
- Be repeated term after term by the same teacher with the same material
- Be from 'consumable' works
 - workbooks, standardized tests, test booklets
- Create or replace course packs



3/1/2010

Off-air Taping Guidelines

- May be recorded and retained by non-profit educational institution for 45 calendar days
- Recordings may be used once by individual teachers, and repeated once for reinforcement, within first 10 days of 45 day retention period
- After 45 days, recordings must be erased or destroyed
- Include original copyright notice



Showing a Film in Class

- Section 110 (1) Exemption for Face-to-face teaching
- Not-for-profit institution
- Film needs to be related to a formal course offering and shown to enrolled students (no clubs, etc.)
- Film must be a legally made and acquired copy



Questions for the Online Classroom

What are the issues if the class is online instead of face-to-face?

- Can I use anything I find on the Web in an online course?
- Is fair Use the same online as it would be in the on-campus classroom?
- Do I need to use an official Course Management System?
 - What about having my own website instead?
 - What about teaching with Google Docs or a similar system?



3/1/2010

Online Copyright Issues

Concerns of copyright holders:

- Works are not just displayed in a classroom, but are uploaded and can be copied
- Ease of online dissemination
- Password protection: any kind of online Fair Use claim also assumes use of a secure Course Management System



Not Favorable to Online (or any) Fair Use

- Repeated or long term use
- Reasonably available licensing
- Affordable and accessible permission
- Posting the information on the free Web



How can Educators Find and Use Third Party Materials?

- Public Domain
- Fair Use
- Library subscription databases
- Websites of free materials
- "Some rights reserved" licenses
 - Creative Commons
- When all else fails—Permission!



3/1/2010

Library Subscription Databases and Licensed Materials

- Magazines
- Journals
- Newspapers
- E-books
- Photographs
- Digital streaming
films



Library Database Persistent Links and Licensed Digital Media

- Licensed subscriptions already paid
- All students and faculty are authorized users
- No permissions costs
- Quality, scholarly resources
- High quality content does not come cheap—
make use of it!
- Contact your librarians for more information



The screenshot shows the MCCCD Libraries website. It features a header with the MCCCD logo and navigation links. Below the header, there are several sections: 'Find a Resource', 'Browse by Discipline', 'Browse by Format', 'Browse by Type', and 'Browse by Subject'. Each section contains a list of links to various databases and resources. On the right side, there is a 'Log In' button and a 'Search' button. The website is designed to help users find and access digital resources.

3/1/2010

Links from the Free Web

- No permission needed for a simple link
- Open link in new window, with original URL showing
- Do not frame linked content within your CMS
- Watch for deep linking
- Do not use a copyrighted image, logo or TM as the hyperlink
- Additional issues with User Generated Content sites (YouTube, Flickr, etc.)



List of Good Free Resources

- Open Courseware
- Public Domain Book Collections
- Image Collections
- Public Domain Images and Music



About What is CC?

Creative Commons is a nonprofit organization dedicated to expanding the range of creative works available to others and to encouraging people to share their work with others. We do this by providing a set of licenses that allow creators to specify which rights they reserve and which they waive.

CC: expanding free, appropriate legal tools

Our licenses give you the freedom to share your work with others, to build on the work of others, and to use the work of others in your own work. We provide a set of licenses that allow you to specify which rights you reserve and which you waive.

Where to find more information

Creative Commons is a nonprofit organization dedicated to expanding the range of creative works available to others and to encouraging people to share their work with others. We do this by providing a set of licenses that allow creators to specify which rights they reserve and which they waive.

CC: expanding free, appropriate legal tools


Our licenses give you the freedom to share your work with others, to build on the work of others, and to use the work of others in your own work. We provide a set of licenses that allow you to specify which rights you reserve and which you waive.

3/1/2010

The Permissions Process


Finding the copyright holder

- Look at your item for copyright information
- Review copyright statement
- Copyright Clearance Center for print works
 - www.copyright.com
- Websites of major publishers
- Website of producer of media
- Music
 - Harry Fox Agency, ASCAP, BMI
- Individual copyright holder
- Secure permission in writing (e-mail is fine)




Copyright Clearance Center

- Pre-cleared permissions from hundreds of publishers
- Institutional or transactional licenses
- Transactional—"pay per use"
- Price based on number of pages and number of copies



The Permissions Process

- Request in writing
 - Your name, address, title
 - Intended use
 - Detailed bibliographic information
 - Number of copies/students
 - Request written reply
 - Allow space for return permission on your letter
 - Ask to forward on if not the copyright holder

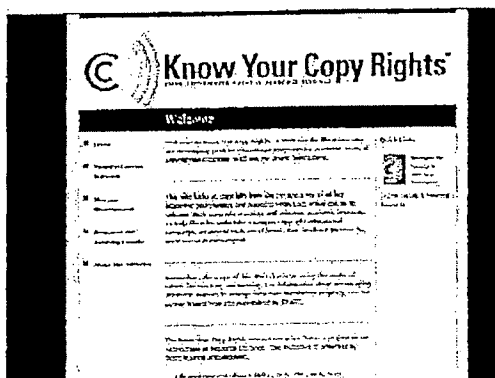


No Response?

- Re-evaluate Fair Use Factor 4
- Limit or change your use to meet Fair Use
 - Use less
 - Password protection
- Decide if "end justifies the means"
- Document all your attempts
- Be prepared to justify what you decide



www.knowyourcopyrights.com



3/1/2010

Best Practices for Faculty

- Be informed; make a GOOD FAITH effort
- Conduct the 4-pronged Fair Use test, using one of the forms provided
- Acknowledge sources of all materials used
- Always include a copyright attribution within your course
- Keep copies of all correspondence
- Document and archive all contacts—telephone calls, etc.
- Negotiate if copyright fees seem too high
- Honor terms of use posted by Copyright owners
- Don't seek permission if it's not necessary, but act from an informed position
- Look for new materials for each new session
- Maximize use of materials licensed by your library



Copyright within MCCCD—the Legal Perspective

Maggie McConnell
Assistant General Counsel



Objectives

- Provide information about MCCCD requirements and resources
- Offer guidance on MCCCD expectations of faculty in considering copyright issues
- Offer some "real life" examples



3/1/2010

MCCCD Requirements

- Federal copyright law requires that MCCCD:
 - Put policies in place regarding copyright and provide information regarding copyright
 - For use of materials on-line:
 - Apply technological measures that reasonably prevent recipients from retaining works beyond a class or further distributing them
 - Refrain from interfering with technological measures taken by copyright owner to prevent retention and distribution



MCCCD Administrative Regulation

3.2 Copyright Regulation

- It is the intent of the Governing Board of the Maricopa County Community College District to adhere to the provisions of the U.S. Copyright Law (Title 17, United States Code Section 101 et seq.). Though there continues to be controversy regarding interpretation of the Copyright Law, this policy represents a sincere effort by the Board to operate legally within the District.



MCCCD Administrative Regulation

- The Governing Board directs the Chancellor or his designee(s) to develop and distribute to employees guidelines that (1) clearly discourage violation of the Copyright Law and (2) inform employees of their rights and responsibilities under the Copyright Law.



3/1/2010

MCCCD Administrative Regulation

- Each college president or provost and the Chancellor shall name an individual(s) at each district location who will assume the responsibilities of distributing copyright guidelines, act as a resource person regarding copyright matter and provide training programs on current copyright laws.
- Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) licenses or contractual agreements, or (4) other permission.



MCCCD Administrative Regulation

- The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.
- In order to assist employees and students in complying with the Copyright Law, appropriate notices shall be placed on or near all equipment capable of duplicating copyrighted materials.



MCCCD Copyright Guidelines

- It's in a user-friendly format
- Go to the following web address to view:
<http://www.maricopa.edu/legal/ip/guidelines.htm>



3/1/2010

MCCCD Copyright Guidelines

- The Guidelines address a full-range of issues:
 - Use of MCCCD resources
 - Copyright basics
 - Fair use principles
 - Getting permission
 - Sample use agreements



MCCCD Copyright Guidelines

- There are also some suggested minimum guidelines for certain types of media or situations:
 - Library uses
 - Printed materials
 - Audio-visual, music and performances
 - Computer software
 - Web site and the internet



Applying the MCCCD Copyright Guidelines and the Law

- The law dictates when and how you can do the following for a copyrighted work:
 - Show it
 - Copy it
 - Perform it
 - Cut and paste it, or copy it, into something else
 - Distribute it
 - Recite it
 - Publish it



3/1/2010

Some Copyright Law Definitions

- "Perform" means "recite, render, play, dance, or act (a work), either directly or by means of any device or process or, in the case of a motion picture or other audiovisual work, to show its images in any sequence or to make the sounds accompanying it audible."



Some Copyright Law Definitions

- "Literary works" are "works, other than audiovisual works, expressed in words, numbers, or other verbal or numerical symbols or indicia, regardless of the nature of the material objects, such as books, periodicals, manuscripts, phonorecords, film, tapes disks, or cards, in which they are embodied."



Some Copyright Law Definitions

- "Display" means "to show a copy of [a work], either directly or by means of a film, slide, television image, or any other device or process or, in the case of a motion picture or other audiovisual work, to show individual images nonsequentially."



3/1/2010

Some Copyright Law Definitions

- "Publicly" means -

- (1) to perform or display [a work] at a place open to the public or at any place where a substantial number of persons outside of a normal circle of a family and its social acquaintances is gathered; or
- (2) to transmit or otherwise communicate a performance or display of the work to a place specified in clause (1) or to the public by means of any device or process, whether the members of the public receive it in the same place or in separate places and at the same time or at different times."



Applying the MCCCDC Copyright Guidelines and the Law

- The MCCCDC Guidelines do not substitute for faculty performing an analysis of the particular use to which a copyrighted work will be put in each situation.
- While lawyers and judges who deal with these matters much more regularly than you do disagree about the law, you are still obligated to perform a "due diligence" analysis about what you are doing, and to retain evidence of your efforts.



Applying the MCCCDC Copyright Guidelines and the Law

- You must always include an attribution to the copyright owner when you copy or otherwise use someone's copyrighted work.
- You must also start with a lawfully purchased copy of the copyrighted work. If you use something that isn't lawfully purchased, it doesn't matter whether a copyright law exception or "fair use" applies. You have violated the law.



3/1/2010

Applying the MCCCDC Copyright Guidelines and the Law

- Read the fine print of the book, DVD or CD that you want to use.
- Books, compilations, or other similar materials are going to contain language similar to the following, which means that you must find an exception under copyright law if you are going to use the excerpts without getting permission:

"All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher."



Applying the MCCCDC Copyright Guidelines and the Law

- If a CD or DVD says, "for personal use only," that is the only authorized use by the copyright holder. Thus, you can use that copy for other uses only with explicit permission or through an exception under copyright law.
- Sales personnel for a publisher/producer aren't going to have the authority to grant you permission to use a work or part of it.



Other MCCCDC Resources

- Legal Services Department web site
<http://www.maricopa.edu/legal/ip/index.htm>
- MCLI Copyright Resource Center
<http://mcli.maricopa.edu/copyright>
- Maricopa Integrated Risk Analysis (MIRA)
<http://www.maricopa.edu/mira/>



3/1/2010

What Licenses Does MCCCC Have?

- Generally speaking, MCCCC has licenses only for limited types of things – periodical databases, music, and computer software.
- MCCCC use of an item licensed under those agreement is carefully circumscribed. The agreements generally don't address any "fair use" issues.



What Licenses Does MCCCC Have?

- For music, MCCCC only has performance licenses with organizations representing music composers and publishers
 - American Society of Composers, Authors and Publishers
 - Broadcast Music, Inc.
 - SESAC, Inc.



What Licenses Does MCCCC Have?

- MCCCC does not have any licenses with performers or the producers of their CD's.
 - That means that we can't use a musical recording without permission or unless the situation qualifies as "fair use"
 - Even with that permission, you still need the permission of the composer and publisher. Our licenses with them don't apply to recordings.



3/1/2010

What Licenses Does MCCCCD Have?

- The licenses with ASCAP, BMI and SESAC (composers and music publishers) are limited.
 - They permit performances only on our premises or sites we engage for performance as part of our non-profit, community outreach and educational activities
 - They do not permit "paid" performances



What Licenses Does MCCCCD Have?

- The licenses with ASCAP, BMI and SESAC (composers and music publishers) are limited.
 - They only permit performances that are "non-dramatic"
 - Excluded are:
 - Music that accompanies dance, dialogue, stage action, visual representation, or pantomime
 - Musical comedy, opera, play with music, revue, or ballet



What Licenses Does MCCCCD Have?

- The licenses with ASCAP, BMI and SESAC (composers and music publishers) are limited.
 - They basically cover performances by faculty, staff, students, and alumni
 - They don't authorize broadcasts, telecasts, or transmission by wire except in some narrow circumstances



3/1/2010

What Licenses Does MCCCCD Have?

- What does this mean?
 - Use of recorded music in a film or video to be shown other than in a bricks and mortar class, even if produced by a student for a class, isn't covered when shown outside of the class
 - Use of recorded music in a dance recital, even if part of a class, isn't covered
 - You need a synchronization or production license for those types of things



What Licenses Does MCCCCD Have?

- What does this mean?
 - Student band performances at an off-campus site and sponsored by a third party, such as shopping center developer or concert promoter
 - Aren't covered where the band is paid as a fundraiser
 - Aren't covered even if payment isn't involved if the activity doesn't qualify as "non-profit, community outreach and educational"



What Licenses Does MCCCCD Have?

- What can you do to protect yourself and MCCCCD?
 - Try to get permission
 - Go through the "fair use" analysis to see whether it applies
 - Do a risk analysis
 - Maricopa Integrated Risk Analysis offers tools for evaluating risks versus benefits
 - See: www.maricopa.edu/mira



3/1/2010

What Licenses Does MCCCC Have?

- MCCCC has many licenses for databases of publications and articles
 - Some require that MCCCC be responsible for the use of all “authorized users,” which includes students
 - We negotiate that term
 - While we won’t agree to be responsible for student misuse, we will promise to inform all users of their copyright obligations



What Licenses Does MCCCC Have?

- MCCCC has many licenses for databases of publications and articles
 - Where the license doesn’t authorize access by staff at non-affiliated libraries participating in collaborative digital reference services with us, we include that language in the license



What Licenses Does MCCCC Have?

- MCCCC has many software licenses
 - They generally preclude reverse engineering or making derivative works
 - Be aware if you are “tweaking” commercial software
 - No matter how great the “tweak” is, we cannot share it with other educational institutions unless the license permits
 - Beware of instances in which you permit non-MCCCC employees to use MCCCC computers (other than the public in libraries)



3/1/2010

What Licenses Does MCCCC Have?

- Be careful about what is recorded and played on MCTV
 - The cable industry has a license with ASCAP, BMI, and SESAC covering music performed on cable
 - It doesn't cover music from CDs, or other types of copyrightable materials (visual, written) displayed on cable



What Licenses Does MCCCC Have?

- Be careful about what is recorded and played on MCTV
 - Even if "fair use" applies to the original use of the work, it may not apply to the broadcast
 - MCTV folks are aware of these issues and can help you figure out the issues



MCCCC War Stories

- In the past 5 years, MCCCC has had to pay about \$260,000 to an author because an MCCCC faculty member placed a digital copy of her work on an unprotected web site.
- Within the last 4 years, MCCCC settled a lawsuit for \$250,000 with a software company that asserted that our continued use of the software after the expiration of the license violated the company's rights.



3/1/2010

Bottom Line

- Being cavalier about copyright raises the potential for serious liability. It's a waste of taxpayers' dollars to have to pay for copyright violations when information or assistance on copyright questions is available.
- You are on the front line daily in keeping MCCCDC out of trouble.
- You don't need to be an expert, but you need to think, analyze and ask questions if you have any doubts about what you are doing.



QUESTIONS?

Maggie McConnell

margaret.mcconnell@domail.maricopa.edu

480-731-8888

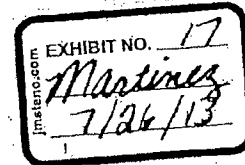
Hazel Davis

Hazel.davis@riosalado.edu

480-517-8273



EXHIBIT 17



VIA HAND-DELIVERY

DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

April 2, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

Effective immediately, your copying privileges at the Phoenix College IKON Copy Center are suspended. The recent incident involving the copying of portions of the book entitled "Recalculus Concepts Through Functions - A Right Triangle Approach to Trigonometry" by Sullivan and Sullivan gives me no other choice but to protect the College's legal interests by imposing this restriction. As detailed below, you knew or should have known that your actions violated federal law and Maricopa County Community College District ("District") regulations. Until I reinstate your copy privileges, all your copying requests must be approved in writing by your department chair in advance.

Respect for the work of others, including proper attribution of authorship and respect for copyright among other ethical standards, is a fundamental tenet of scholarship. I find it impossible to believe that a person who has been awarded a doctorate is so unfamiliar with the applicable legal and ethical standards. Moreover, during your long career with Phoenix College and the Maricopa County Community College District, you received extensive information about District Administrative Regulation 3.2 concerning copyright law and fair use guidelines as well as information about the federal Copyright Act. Administrative Regulation 3.2, which may be found on page 42 of the Blue Book, which is prominently available on the Legal Services Department's website as are the District's Administrative Regulations, of which that regulation is a part. All District employees were e-mailed a notice of the availability of the Blue Book on January 6, 2009 and asked to bookmark and acknowledge it. Administrative Regulation 3.2 reads as follows:

3.2 Copyright Regulation

1. *It is the intent of the Governing Board of the Maricopa County Community College District to adhere to the provisions of the U.S. Copyright Law (Title 17, United States Code Section 101 et seq.). Though there continues to be controversy regarding interpretation of the Copyright Law, this policy represents a sincere effort by the Board to operate legally within the District.*
2. *The Governing Board directs the Chancellor or his designee(s) to develop and distribute to employees guidelines that (1) clearly discourage violation of the Copyright Law and (2) inform employees of their rights and responsibilities under the Copyright Law.*
3. *Each college president or provost and the Chancellor shall name an individual(s) at each district location who will assume the responsibilities of distributing copyright guidelines, act as a resource person regarding copyright matter and provide training programs on current copyright laws.*

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Dr. Cleopatria Martinez

April 2, 2010

Page 2

4. *Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) Licenses or contractual agreements, or (4) other permission.*
5. *The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.*
6. *In order to assist employees and students in complying with the Copyright Law, appropriate notices shall be placed on or near all equipment capable of duplicating copyrighted materials.*

Moreover, not long before the recent incident arose, Phoenix College officials including Ronnie Elliott, Paul DeRose, and Casandra Kakar as well as District legal counsel Maggie McConnell extensively discussed with you the requirements of the Administrative Regulation. Finally, Phoenix College officials encouraged you to attend the March 1, 2010 Phoenix College workshop to learn more about copyright/fair use. However, you chose not to attend the workshop.

You are reminded that Section 4 of Administrative Regulation 3.2, quoted above, clearly advises that the Governing Board prohibits and disapproves of unauthorized duplication in any form. Section 5 of the Regulation further advises that employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

You are finally admonished that any further copyright/fair use violations will lead to disciplinary action being taken against you, up to and including termination of your employment.

I am willing to arrange for individual training and consulting on copyright if you feel it would be helpful. But I will restore your copying privileges only when I determine that you not only understand your legal obligations, but also that you will strictly adhere to them.

Sincerely,



Anna Solley, Ed.D.
President

cc: Mr. Paul DeRose
Mr. Richard Galvan
Ms. Casandra Kakar
Ms. Maggie McConnell

EEOC000192

EXHIBIT 18



DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

VIA HANDDELIVERY

October 12, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez:

Given our recent meeting as per the RFP Section 6.1.2.2, I am writing in response to the grievance you filed which indicated that Department Chair Joe Sueyoshi violated Section 3.1. of the RFP by denying you the right to determine appropriate teaching materials relevant to your subject. I support Mr. Sueyoshi's and Vice President for Academic Affairs Casandra Kakar's decision to deny printing or copying of your work which continues to remain in question concerning violation of copyright and fair use laws.

The conflict over copying first arose after you created course materials that were derived from the required text or other sources without permission from or attribution to the author or publisher. You borrowed text and example problems and called these "lecture notes", copied them and made them available to students for free. You also borrowed without permission or attribution test problems from the required text. Students learned that they did not need to buy the text for your course and several returned the text for a full refund.

The following chronology clearly delineates the growing concerns about your failure to understand and follow copyright and fair use law and your professional obligation to attribute borrowed material to the author and publisher:

1/5/10 – VPAS Ronnie Elliott and VPAA Casandra Kakar were notified of suspected copyright violations concerning Dr. Cleopatria Martinez.

1/7/10 – VPAS Ronnie Elliott and VPAA Casandra Kakar met with district legal counsel and presented the materials to Richard Galvan and Maggie McConnell.

1/8/10 – VPAS Ronnie Elliott notified VPAA Casandra Kakar of approximately thirty printing requests by Dr. Cleopatria Martinez. They contained materials that PC administrators had alerted district about the previous day.

1/12/10 – VPAS Ronnie Elliott notified Dr. Cleopatria Martinez that mechanisms in place at PC had red flagged items she printed in Fall 2009 and Spring 2010. Ms. Elliott asked for documentation that verified approval of the use from the publisher. Dr. Martinez was informed that until proper documentation was provided her print jobs would not be allowed.

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MCCCD/Martinez00968

Cleopatria Martinez

10/12/10

Page 2

1/14/10 - Dr. Cleopatria Martinez could not remember what she had printed for the fall semester. VPAS Ronnie Elliott responded that the classes were MAT182 and MAT187. Dr. Martinez felt Ms. Elliott's email was ambiguous and wanted specific pages.

1/15/10 - VPAS Ronnie Elliott responded that the entire documents were in question. Once again she asked Dr. Martinez for documentation of the permission from the publisher.

1/20/10 - Dr. Martinez challenged Ms. Elliott and wanted more information regarding the mechanisms that prompted the concerns about copyright. Dr. Martinez found it hard to believe the entire document was a concern. She also indicated that she would be reporting concerns to Dr. Solley because the hold on materials would impede her ability to teach her classes.

1/22/10 - Margaret McConnell notified Ms. Elliott in writing that the issue needed to be resolved quickly because her review of the materials indicated copyright violations and possible plagiarism. Ms. McConnell wrote a detailed summary of her concerns in the email.

1/26/10 - VPAS Ronnie Elliott sent an email message to Dr. Martinez outlining Maggie McConnell's concerns about copyright infringement and fair use as defined in federal law due to Dr. Martinez's recent copying requests. Ms. Elliott wrote: *"In light of those legal standards, we have reviewed your January 20 e-mail and the following materials - a document entitled 'Phoenix College Fall 2009, MAT 182 Trigonometry Section, Lecture Notes, Dr. Cleopatria Martinez' and a book, Precalculus: Concepts Through Functions: A Right Triangle Approach to Trigonometry, Instructor's Edition, authored by Michael Sullivan and Michael Sullivan, III. The book indicates that the copyright holder is Pearson Education, Inc. and it says: 'All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.' The first issue here is whether your lecture notes are original works or problems created by you, or whether portions are instead copied from the Precalculus book or some other copyrighted source. In reviewing parts of your notes and the book, we have some real concerns. Attached are some pages of your notes on which we have handwritten the page numbers of the Precalculus book as the source of the example, text or problem. Once we came up with these 10 or so correlations, we stopped looking, as the problem seemed clear. The examples or problems are copied verbatim into the lecture notes. Additionally, there is no attribution in each of these instances of the source of the example or problem, that is, no identification of Pearson as the copyright holder. Whether you intend to or not, the lecture notes give the distinct impression that they are your original work, which is incorrect. That lack of attribution is, by itself, a legal issue. As to what the breadth of the problem is, only you know that because we don't know all of the resources that you may have used for those notes. Moreover, it is our understanding that you may have been using these notes over a period of time. As you can see from the listing of the 'fair use' requirements, the use must not be one that, if it became widespread over an entire market, would reduce the potential market for the copyrighted work. In this case, copying problem or examples from the book, with the potential that the students don't need to purchase it, would make your use without permission a problem. Other guidelines developed under the auspices of the federal government specify that use must 'spontaneous.' It is hard to make a case that this is spontaneous use if you use the materials over and over each semester. In any event, it would appear to have taken a significant effort to copy the problems and examples into the text of your lecture notes, thereby eliminating any 'spontaneity.' Based on the above, we believe that you need to get written permission to use portions of the book in your lecture notes from Pearson, and make sure that you note that Pearson is the copyright holder of materials. Publishers such as Pearson have specific staff dedicated to this issue. You should find out who those are, identify the portions of the book that you wish to use, and obtain permission in writing. Bottom line, Cleopatria, you must secure written permission from the publisher before we are able to print your material."*

Cleopatria Martinez

10/12/10

Page 3

1/28/10 - Dr. Martinez contacted Margaret McConnell regarding copyright. Ms. McConnell documented the conversation and sent a written notice to Dr. Martinez explaining in detail information about copyright issues and resources.

1/29/10 - IKON contacted Paul DeRose regarding a print job Dr. Martinez was requesting. IKON was told by Ms. Elliott not to print Dr. Martinez's print jobs without administration's permission.

2/3/10 - VPAA Casandra Kakar responded to Dr. Martinez's 1/28/10 memo and indicated that her materials were being reviewed and a response would be provided the next day.

2/5/10 - VPAA Casandra Kakar and Interim VPAS Paul DeRose met with Dr. Martinez and her faculty representative, Dennis Sheehan, to discuss copyright concerns. Listed below is a summary of the meeting from 2/5/10 that was sent to Dr. Martinez on 2/12/10:

Present: Casandra Kakar, Paul DeRose, Cleopatria Martinez, Denny Sheehan

Summary: Copyright laws were discussed. The copyright verbiage is printed in the front of the textbook that is being used in the Math class. In order to use material from the text, prior approval must be obtained from the Publisher's Legal department. An approval from the Publisher's rep is not acceptable. Cleopatria was referred to the email sent by Maggie McConnell which describes the copyright issue and contains a link to seek Publisher approval. Cleopatria said she understands the copyright law and will not use materials from any textbook without prior approval from the Publisher's Legal office. If Cleopatria decides to bind her notes/handouts, Casandra recommends that they be distributed by the bookstore - similar to the practice used by other faculty in her department and at Phoenix College. This will be a nominal fee to the students, and will also minimize costs to the department's print budget. The posting of notes on Blackboard is also an option. In addition to our discussion, we would also like to encourage you to attend an upcoming Copyright Workshop that is being held at Phoenix College. This workshop is scheduled for March 1st from 1:00pm - 3:00pm.

2/5/10 - As a result of the meeting and Dr. Martinez's agreement to not violate copyright, Interim VPAS Paul DeRose notified IKON to release the hold on her print requests.

2/12/10 - Summary of meeting was sent to Dr. Martinez by Paul DeRose.

3/1/10 - Copyright workshop was held at Phoenix College. 29 faculty/staff attended and Dr. Martinez did not attend.

3/5/10 - Math Department Chair Joe Sueyoshi notified VPAA Casandra Kakar that Dr. Martinez printed 31 copies of a 24-page document which was from the materials she tried to use at the beginning of the semester for her Trigonometry class. Mr. Sueyoshi was concerned about copyright and curious about why she printed 700+ pages when students supposedly had purchased textbooks for the class.

3/9/10 - Math Department Chair Joe Sueyoshi notified Interim VPAS Paul DeRose and VPAA Casandra Kakar of his concerns regarding Dr. Martinez's print jobs. He indicated that Dr. Martinez was picking up all her print jobs directly from IKON in order for the department not to see the contents of copyright violation materials.

Cleopatria Martinez

10/12/10

Page 4

4/2/10 – PC President Dr. Solley notified Dr. Martinez in writing that her copying privileges were suspended and all copy jobs must be approved in writing by the department chair due to concerns about the recent incident involving her copying of portions of the book entitled “Precalculus Concepts Through Functions – A Right Triangle Approach to Trigonometry” by Sullivan and Sullivan without permission from the publisher or attribution to the authors. Dr. Solley reminded Dr. Martinez about MCCD’s Administrative Regulation 3.2. on Copyright Regulation specifically Section 4 which clearly advises that the Governing Board prohibits and disapproves of unauthorized duplication in any form. Dr. Solley also informed Dr. Martinez that she was willing to arrange for individual training and consulting on copyright and that Dr. Martinez’s copyright privileges would be restored only when Dr. Solley determined that Dr. Martinez not only understood her legal obligations but also strictly adhered to them.

4/8/10 - Margaret McConnell sent an email message to Dr. Martinez to document a conversation that she had with her. Dr. Martinez was seeking approval to create a derivative work that amounted to a “reader’s digest” version of the text to be published under her own name, that was intended to supplant the text not interpret or explain it. Once again, Ms. McConnell explained to Dr. Martinez that her actions were still considered a violation of copyright law. Ms. McConnell wrote: *“In fact, no MCCCD employee is going to be in a position to tell you that ‘tweaking’ is OK, particularly when the owner of the original work isn’t given credit for the ‘tweaked’ work. The verdict under the law would almost certainly be that ‘tweaking’ would not solve the problem, and that is the answer that you are going to get from a wide range of folks that you might consult. Changes to an exiting copyrighted work make the work a derivative under the law, and the copyright owner of the original work has rights in that changed work as well as to the original. It appears from our conversation that you persist in thinking, however, that ‘tweaking’ at a certain point will be OK from a copyright standpoint. That is troublesome, given that the college and, to some degree, this office have been dealing with you on this issue at a minimum for nearly two months. The more that you push to find a bright line between a copyright violation and changing someone’s work to make it appear to be original, which does not exist in the law, the more it will be difficult for your department and administration to feel comfortable with giving you copying privileges. You may avoid all of the questions and potential legal pitfalls by simply crafting all of the illustrative equations in your lecture notes yourself. I have received a copy of the letter that Dr. Solley recently sent you concerning these matters and given the potential for personal liability on them in the future, I would urge you to create your own equations. Based on the observations I make in this note, it will not be useful for us to meet and go over your materials.”*

4/8/10 - District Legal Counsel Lee Combs followed up with another email attempt to convey the serious implications that the actions Dr. Martinez continued to take constituted a violation of law. Mr. Combs wrote: *“Our advice is that the course materials Dr. Martinez distributes to classes must be clearly and demonstrably her own original work, unless she attributes authorship to another, whose permission for such use is documented.”*

4/15/10 - PC Librarian Ann Roselle held a one-on-one copyright training session with Dr. Martinez and discussed the same copyright and fair use laws that were presented in the workshop. Dr. Martinez also received copies of the handouts from the workshop. In addition, they discussed some of her instructional materials within the context of copyright and fair use.

4/19/10 - Dr. Martinez had an adjunct faculty math instructor print 24 copies of a document. The document had not been reviewed by the department chair.

Cleopatria Martinez

10/12/10

Page 5

4/19/10 - In addition, the same morning Dr. Martinez made at least 14 copies of a 51-page document - her MAT182 notes, using the math department laser printer. Two eye-witnesses observed the department laser printer going non-stop.

4/20/10 - A student of Dr. Martinez indicated to the department chair that Dr. Martinez did not use the text listed in her syllabus and was using the trigonometry notes as originally created at the beginning of the semester. These are the notes that were not allowed to be used.

4/22/10 - Math Department Chair Joe Sueyoshi declined to print an outline and 50 page document because of the copyright and possible plagiarism issues.

4/23/10 - Dr. Solley notified Dr. Martinez of the Administrative Evaluation that would be conducted as per the RFP Section 3.7 due to Department Chair Joe Sueyoshi's concerns about violation of copyright laws, plagiarism of Trigonometry (MAT 182) class notes, insubordination, and abuse of math department resources (paper, cartridge ink, and printer).

5/10 - District Legal Counsel Lee Combs and Dr. Martinez's attorney agreed that the Administrative Evaluation and Dr. Martinez's grievance would resume in August, 2010 due to summer vacation.

8/17/10 - Math Department Chair Joe Sueyoshi approved request from Dr. Martinez to print syllabus and personal information sheet for fall classes.

8/18/10 - Mr. Sueyoshi received a request to print 32 copies of 115 page MAT082 materials. He was not confident that all contents were Dr. Martinez's original work. In addition, the materials should be sold through the bookstore as recommended in the meeting with her on 2/5/10.

8/19/10 - Mr. Sueyoshi declined the print job for the following reasons:

1. Cost to the department for binding.
2. Course competency seven not met.
3. Concerns that images and word problems were not original - potential Federal Copyright violations.

8/19/10 - Dr. Martinez request that the MAT187 notes be copied.

8/20/10 - Mr. Sueyoshi declined: *"I cannot in good conscience approve this request until you address my concerns. My rationale is based on the following details:*

(1) These algebra notes, which you want printed, are basic equivalent to the first 5 chapters of the Sullivan text, the text listed for use in your syllabus.

(2) According to your Permission from Pearson to make copies from Chapter 6, 7, 8, & 9, 'Permission is granted free of fee on the understanding that the above textbook (Sullivan) has been adopted for the course and purchased by the students.'

It appears, based on (1), that you plan NOT to use the Sullivan text for Chapters 1-5. Do you plan to use the Sullivan text for Chapters 6-9? Or do you plan to use your previously developed MAT182 notes? In any case, I have concerns here. Please respond, Joe"

Cleopatria Martinez

10/12/10

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8/20/10 - A conference call was held by District Legal Counsel Lee Combs and Dr. Martinez's attorney. Dr. Solley, VPAA Casandra Kakar and Dr. Martinez participated in the meeting. The following items were agreed upon:

1. *Dr. Martinez would confirm in writing to Dr. Solley that she would announce to her class that they must purchase the textbook for MAT187.*
2. *The first twenty pages of MAT082 product would be printed for her class, stapled and in black and white.*
3. *One copy of MAT187 will be copied for Dr. Martinez's personal use only. It is not to be distributed or printed by Dr. Martinez. Dr. Solley informed Dr. Martinez that we are not confident that the notes which she wants to distribute are original works. Until the notes are approved by the department chair, we must protect the college and district. We will not break copyright laws.*
4. *The committee of faculty members that reviewed the content of MAT082 will be provided with a copy of the document. The committee is to work together and review materials and determine if contents are original.*

8/21/10 - Email exchanges with Dean for Academic Affairs Wilbert Nelson, Mr. Sueyoshi and Dr. Martinez. All MAT 082 items noted in the attorney meeting on 8/20/10 were delivered on 8/21/10.

8/21/10 - Dr. Martinez wanted to know if the MAT187 notes were approved. Mr. Sueyoshi indicated that she is only to receive one personal copy. He also let her know that since the students are purchasing the textbook there is no need for the students to have the notes. She asks if he will approve certain pages of the notes for distribution.

8/23/10 - Mr. Sueyoshi refers her back to his message from 8/20/10 and declines her request. Dr. Martinez asks Mr. Sueyoshi to print nine pages which included five pages of graphs from the MAT187 document within minutes of her class meeting time. He agreed to print four of the nine pages that we not from MAT187 document.

8/23/10 - Dr. Solley met with the members of the Administrative Evaluation Team to discuss the Administrative Evaluation that would be conducted as per the RFP Section 3.7 due to Mr. Sueyoshi's concerns about violation of copyright laws, plagiarism of Trigonometry (MAT 182) class notes, insubordination, and abuse of math department resources (paper, cartridge ink, and printer).

8/24/10 - Dr. Martinez asks Mr. Sueyoshi to print a copy of the college directory. He declines because the college has not printed for over two years. The college has gone green and we are all to use the electronic directory. In addition, he notes that the current directory has not been updated for a number of years. The most accurate information is found on the memo system. Administration found an old, unused copy of the last printing of the directory. It was offered to Dr. Martinez for her use.

8/26/10 - VPAA Casandra Kakar and District Legal Counsel Lee Combs met with Dr. Martinez and her attorney regarding her grievance about her concerns that her academic freedom was being violated. It was agreed that Dr. Martinez never completed the informal grievance step and that she must follow the process and meet with Mr. Sueyoshi to discuss and try to resolve. At this meeting, she did inform the meeting participants that she had spent the entire summer rewriting the MAT187 notes that she wanted copied for distribution. Since the notes were now her original work, she did not ask the publisher to approve usage for Chapters 1 - 5. She did get permission from the publisher for Chapters 6-9, but there was a stipulation that the students that received her notes must purchase the textbook.

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8/30/10 – Dr. Martinez and Mr. Sueyoshi met to resolve the grievance at the informal level. However, Mr. Sueyoshi noted that she never indicated to him that her academic freedom was being violated. Rather she wanted him to show her where her copyright violations were. He very quickly pointed out two in the first few pages.

9/7/10 - Dr. Martinez requested that VPAA Kakar approve five pages of graphs be printed for her class. VPAA Kakar declined because they were the same five pages of graphs that Mr. Sueyoshi was asked to print on 8/23/10.

9/9/10 - Dr. Martinez asked for 35 copies of a one page outline from MAT187 be copied for her class. The outline included page numbers from the MAT187 document. Mr. Sueyoshi declined because there was no use for it unless she had distributed the notes. If she had not distributed the notes, the outline would confuse the students.

9/10/10 - Dr. Martinez involved the department secretary and asked him to modify the document by removing page numbers.

9/10/10 – Mr. Sueyoshi sent options to Dr. Martinez regarding other methods to get the information to the students without using the outline.

9/13/10 – Mr. Sueyoshi's letter of response to the August 30, 2010 meeting with Dr. Martinez was sent and received by all intended parties. He addressed the questions that Dr. Martinez raised at the 8/30/10 meeting. He also asked Dr. Martinez several questions about the documents that had not been approved for printing—to date, Dr. Martinez has not addressed Mr. Sueyoshi's questions. Mr. Sueyoshi concluded the letter by stating: *"In conclusion, it is not my job to correct your work so that it can be approved. It is your job to restore my confidence in your willingness and ability to create materials that give proper attribution to the work of others and that are manifestly free of material borrowed without permission. If you believe that borrowed material is 'fair use', I expect you to identify all the borrowed material and spell out your arguments as such. Because an essential element of 'fair use' is that the use does not undermine the market value of the copyrighted material, I also expect that all students in your class will purchase the required text at the beginning of the semester and not return it until the end of the semester. Your misrepresentations in our meeting did nothing to restore the confidence I need to approve your current copy requests."*

9/15/10 - Dr. Martinez requested that the outline be printed again.

9/19/10 – Mr. Sueyoshi reminded Dr. Martinez that she had not answered the questions about the page numbers on the outline.

9/20/10 - Dr. Martinez requested a print job for an answer key for worksheets that were not approved by Mr. Sueyoshi earlier in the semester. This is a possible indication that she had made improper copies of the worksheets because it was not clear why she needed answer keys.

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9/21/10 - Dr. Martinez and VPAA Kakar met regarding the grievance, now at a formal level. During the meeting, Dr. Martinez wanted approval to print certain jobs. VP Kakar responded:

"Dr. Martinez, Regarding your print request during our meeting earlier today:

- a. Regarding your request for MAT082 - Joe has agreed to approve printing of future homework assignments that contain numbers only. Any homework with assignments with images or words will not be approved as per the letter of the August 30, 2010, meeting.*
- b. The one page document/outline lists section numbers for materials that students do not have, according to you. They reflect the sections of the document that is of concern to our legal department at district, your department chair and me. Since each student has a textbook this should not be an issue.*
- c. Joe has indicated to me per the letter of the August 30, 2010, meeting he cannot in good conscience approve MAT187, pages 4, 5, 11, 12 & 13. The graph/grid images come from multiple sources."*

9/21/10 - Mr. Sueyoshi responded to a request from Dr. Martinez to print a test for MAT187. He declined and provided the following explanation to her:

"It's quite clear that the exam (you have requested to be printed) is aligned with your MAT187 notes and not the textbook. You stated that the students did not have the MAT187 materials. So, how are the students preparing for the exam without copies of your notes? The list of topics covered on your exam are scattered in the textbook (chapters 1, 2, & 10 at least). You even state on the exam that it covers sections 1.1-1.10, which does not correspond to the textbook, but very well to your notes. If your students have your notes in hand already it would also explain why you requested the MAT187 outline printed with page numbers and sections listed."

9/22/10 - Mr. Sueyoshi wrote to VPAA Kakar and Mr. Combs: *"I've attached the exam for the both of you. She returned the exam to me to be 'OK' for printing by physically blacking out the '1.1-1.10' and 'Fall 2009' (she did not delete it electronically for whatever reason). The exam itself is unusual only because it appears she uses different fonts & font sizes on questions #13 & #14 (a sign that the questions were copied, pasted, & edited from another source). The equations themselves can be written using 'MathType', a common software. I don't object to the exam as much as the fact of what the exam indicates: Students are using her MAT187 materials exclusively.*

Although she will claim it was just a mistake, the 'Fall 2009' note indicates that this exam and therefore her MAT187 materials were used last Fall, 2009. Although I have some of her MAT187 materials from Fall, 2009, only an interview with her MAT187 class from Fall, 2009, can confirm she did use this exam a year ago and therefore her materials."

9/23/10 - Dr. Martinez again asked about the following copy requests:

- (1) 1-page outline of her MAT187 materials.
- (2) MAT187 Test (based on her lecture notes, not the text)
- (3) Answer Key to worksheets that were previously denied.

Mr. Sueyoshi's response: *"Concerning the following copy requests:*

- (1) 1-page outline of your MAT187 materials.*
- (2) MAT187 Test (aligns with your MAT187 lecture notes, not the text)*
- (3) Answer Key (to worksheets that were previously denied copy requests).*

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I've been advised to ask the following: Since you stated that you have not been distributing your MAT187 notes - about which concerns have not been resolved, these copy jobs appear to be contrary to this statement. Please justify the request(s) by explaining exactly how you are using your notes to teach this class and how these documents relate to material covered.

Thanks, Joe"

9/27/10 - Dr. Martinez requested that trigonometry notes be copied. Mr. Sueyhoshi declined and responded: *"I must decline this request since it contains graphs from multiple sources for which you have not acknowledged. If you did create them yourself, please explain why you needed different graphing software packages to create the graphs. It appears from the graphs themselves that they are from at least three different sources. If you can name the 3 different software packages, it would be a start. As a footnote, this page is also one of the copy jobs you had an adjunct improperly make last Spring '10. It is also a page taken from your Trigonometry notes, which is involved in the on-going Administrative Evaluation. So, I don't think I can approve any of your Trigonometry notes until the Evaluation is complete."*

9/27/10 - VPAA Kakar responded to Dr. Martinez in writing to the grievance:

"Dr. Martinez, I have reviewed the letter dated September 13, 2010 that Joe Sueyoshi prepared in response to your meeting with him on August 30, 2010 to resolve your grievance. Per the RFP, 6.1.2.1 - Level 1, we discussed your concerns about his decision on September 21, 2010. My decision is to support your Department Chair, Joe Sueyoshi and his decision not to approve the printing or copying of your work that continues to remain in question."

9/29/10 - Mr. Sueyoshi had more concerns regarding requests to print materials in question. His comments to Dr. Martinez follow:

"Did you assign any homework from the assigned text, namely from chapters 1, 2, 10 & section A.9? These are the chapters/sections that are covered on the exam that you have requested to be printed. You also wrote on the exam, before correcting it, that it covered sections 1.1-1.10, which describe your notes, not the assigned text. If you did assign homework from the text, then there is no need for homework to be contained in your notes--your notes, in your words, are just an 'outline' after all. Since the students have the text, they don't have to copy any homework problems. Your notes don't need to be copied since the text contains all the pertinent information. You can direct them, accordingly. May I suggest guided reading assignments so they can create their own notes instead of copying your notes. Also, if you use the padcam, you don't have to write anything on the board. If you did not assign any homework from the text, even though the students are supposed to have the text, then why would students buy the text? You may be suggesting to students that the text is unnecessary if you only use your notes. Your current notes also do not cover all the competencies. There is also, in the response letter from the August 30, 2010, letter, the same concern about 'fair us', etc., especially if the students are not purchasing the text as a result of not assigning homework from the text. I hope you understand my concerns."

9/27/10 -- District Legal Counsel Lee Combs responded to a letter from Dr. Martinez's attorney which threatened legal action. Mr. Combs stated in the letter: *"Since the issue of copyright arose, your client has done absolutely nothing to build my client's confidence that she understands her obligations under copyright law, and her professional obligation to attribute borrowed material to the author and publisher. Instead, she told Mr. Sueyoshi and Ms. Kakar that she created by herself all the materials for which she now seeks copying approval. Mr. Sueyoshi easily exposed that as a falsehood. As we discussed, this is a trust issue. Your client's response to my client's legitimate concerns has been to engage in legal posturing and to lie. If you think you have a case, bring it. Here's a thought that might resolve this matter short of that, once and for all."*

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Dr. Martinez told Ms. Kakar in our presence that she contacted the publisher. She said she was told that her materials were not a problem for them and her borrowings were considered fair use. If you will, provide me the name of the person she contacted. If we can verify that the publisher approves the use of these materials, fully cognizant of the way they were developed and their evident impact on book sales, my client can have no objection."

10/5/10 - Dr. Solley met with Dr. Martinez to discuss the grievance at Level 2.

In summary, at this point I do not have the confidence I need in you to lift the copy restriction I placed on you or to change Mr. Sueyoshi's decisions. I support Mr. Sueyoshi's and Ms. Kakar's decision to deny printing or copying of your work so long as it remains in question concerning violation of copyright and fair use laws. RFP Section 3.1 concerning Instructional Rights/Academic Freedom stipulates that faculty will recommend the appropriate pedagogy and textbooks and other materials relevant to teaching their subject. RFP Section 3.1 does **not** stipulate that faculty can violate copyright and fair use laws. As I indicated to you on April 2, 2010 and during our recent meeting, respect for the work of others including proper attribution of authorship and respect for copyright among other ethical standards, is a fundamental tenet of scholarship. It is your responsibility to follow copyright and fair use laws and to ensure that documents are original works. If these documents are not original works, then it is your responsibility to acquire the publisher's approval or to appropriately attribute the source for the materials.

To assist Mr. Sueyoshi and the Administrative Evaluation Team to resolve the lingering questions about your work, the district has retained expert legal counsel. I am advised that this legal review should be completed by October 22, 2010. In the meantime, it is reasonable that we have controls in place to protect the legal interests of the college and district by restricting your copying and printing privileges due to the loss of confidence that your materials are compliant with copyright and fair use laws. I will restore your copying and printing privileges only when I am confident that you not only understand your legal obligations but also that you will strictly adhere to them.

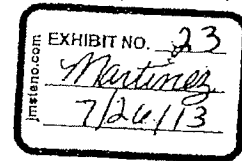
Sincerely,



Anna Solley, Ed.D.
President

cc: Mr. Eddie Genna, Faculty Senate President

EXHIBIT 19



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Confidential Memorandum

October 28, 2010

Our File Number: 31504-00002

To:

Lee Combs
Casandra Kakar

From:

Sean D. Garrison
Nathaniel W. Edwards

Re: Copyright Analysis of Course Materials of Dr. Cleopatria Martinez

EXECUTIVE SUMMARY

Dr. Cleopatria Martinez has prepared a series of course packets (the "Course Materials") for students to use in her mathematics courses. Specifically, we have reviewed the following:

- MAT 182 Precalculus Trigonometry Section Lecture Notes Fall 2009
- MAT 182 Trigonometry Spring 2010 (50 pages obtained from a class student)
- MAT 082 Chapters for Basic Arithmetic Fall 2010
- MAT 187 Precalculus Algebra Section Lecture Notes Fall 2010

None of the Course Materials includes any attribution to other source texts, which indicates that Dr. Martinez is portraying these works as her own independent creations. In the MAT 082 and MAT 187 notes document, Dr. Martinez does generically thank a list of individuals for their inspiration, ideas, comments and suggestions.

Phoenix College asked for our analysis as to whether the Course Materials appear to have been independently created or copied from other sources. Our review of the materials suggests that at least significant portions of three of these Course Materials have been copied from other sources, including *Basic Mathematics* by Charles P. McKeague; *Prealgebra* by Alan R. Tussy and R. David Gustafson; and *Precalculus* by Michael Sullivan and Michael Sullivan, III. Accordingly, we do not recommend that Phoenix College allow further distribution or use of the Course Materials and recommend that the College remove the Basic Arithmetic course document from its website.¹

COPYRIGHT LAW BACKGROUND

A party may obtain copyright protection for "original works of authorship fixed in any tangible medium of expression." 17 U.S.C. § 102(a). Only works of authorship that are original are protected by copyright law. *Id.* "Original, as the term is used in copyright, means only that

¹ We located this packet at the following address:

<http://www.pc.maricopa.edu/data/GlobalFiles/file/mathematics/Math%20Faculty/Cleopatria%20Martinez/Math%20082/082%20Basic%20Arithmetic%20Spg%202010.pdf>



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the work was independently created...and that it possesses at least some minimal degree of creativity." *Feist Publications, Inc. v. Rural Tel. Service Co., Inc.*, 499 U.S. 340, 356 (1991).

Facts, including mathematic definitions, formulas and calculations are not protectable by copyright. See *Educational Testing Services v. Katzman*, 793 F.2d 533, 536-37 (3d Cir. 1986). Although facts are not subject to copyright protection, a "compilation" of facts may be entitled to protection. *Id.*; 17 U.S.C. §103(a). But, that protection extends only to the original material added by the author of the compilation, not to facts contained within the compilation. In other words, a single mathematic calculation is not entitled to copyright protection, but a compilation of calculations may be.

Although based on underlying unprotected facts, word problems and exam questions can be protected expression if they possess "some minimal degree of creativity." See *Feist Publications*, 499 U.S. at 356. Mathematic word problems are protectable where they do not represent the only means of expressing the equations/calculations on which they are based. *Educational Testing Service*, 793 F.2d at 540. In other words, the mere fact that word problems refer to mathematic equations or facts - or require use of equations or facts to solve them - does not, by itself, deprive the problems of copyright protection. See *id.* (finding that there are numerous means to frame questions to test students knowledge of "square roots or dangling participles" and that questions testing this knowledge were protected under copyright law). Therefore, because word problems can reflect original expression in their word choice and explanations, they can be protected by copyright even if they test knowledge of established mathematic formulas.

Copyright infringement can occur even if the amount of material infringed upon is low if the copied material is the core of the protected material. *Harper & Row Publishing, Inc. v. Nation Entertainment*, 471 U.S. 539, 565 (1985). Merely replacing or adding terms or changing numbers in word problems copied from other sources does not necessarily remove the risk of infringement.

Finally, as discussed, Phoenix College and the District could be liable for the infringement of Dr. Martínez even though they did not previously know of or sanction the infringement.

METHODOLOGY

To assess whether the Course Materials were copied from other sources, we reviewed the Course Materials for discrepancies in font and word usage. We also took sections of text from problems and examples in the materials and ran Internet searches for the text. When the text from the Course Materials was found in another source, that source was noted and searched for other portions of the Course Materials. In addition to performing our own searches, we reviewed the notes made by Mr. Joe Sueyoshi regarding the Course Materials and corresponding textbook material.

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We also performed Internet searches for many of the images contained in the Course Materials. But, other than the image of the circle on pages 61 and 72 of the Basic Arithmetic materials and the cylinder on pages 65 and 76 of the Basic Arithmetic materials, brought to our attention by Mr. Joe Sueyoshi in our initial meeting, our searches did not identify any other images in the Course Materials except for one image associated with a word problem. Additionally, we believe that most of the images in the Course Materials - at least by themselves - lack the requisite originality for copyright protection.

Due to time limitations and the limitations of Internet search engines, we did not search for individual equations contained in the materials.

Reviewing mathematics materials for infringement risks presents unique challenges. First, there are countless mathematics textbooks, many of which are not available online or out-of-print. Second, for textbooks that are available online, pages, sections and whole chapters are often removed. Third, it is not practical to search for every problem, example, image or equation contained in the Course Materials or to identify each and every example of copying that we found. Accordingly, just because content from the Course Materials that is not identified in the charts below does not mean that it was independently created.

RESULTS OF COURSE MATERIALS REVIEW

Presented below are charts detailing discrepancies in the text of the Course Materials and instances where we found text similar or identical to the text of the Course Materials in other sources.

Precalculus Trigonometry Section Fall 2009 (MAT 182)

Although the cover page states "Fall 2009," the heading on the internal pages shows Spring 2010. These Lecture Notes were intentionally and systematically copied from the Sullivan book *Precalculus, Concepts Through Functions*, Instructor's Edition. It is our understanding, however, that this textbook was not used in the Spring 2010 class.

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
p. 6	Problems in Section 1.2 copied directly from source text	<i>Precalculus</i> , Sullivan, p. 371-372
p. 8	Example for Area of a Section is copied from source text.	<i>Precalculus</i> , Sullivan, p. 369 (Example 7)
p. 10	Arc length problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 372
p. 14	Right Triangle homework problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 383-384
p. 16	Example 6 copied from source text, no examples 1-5 appear in this section of the course materials	<i>Precalculus</i> , Sullivan, p. 382
p. 17	Pythagorean/identity problems are copied from source text	<i>Precalculus</i> , Sullivan, p. 384

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COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
p. 20-21	Examples 1-5 are identical to source text	<i>Precalculus</i> , Sullivan, p. 387-390
p. 22	Problems (including introductory text) copied from source text; "Projectile Motion" section (including the image) is copied verbatim from source text	<i>Precalculus</i> , Sullivan, p. 393-394
p. 23	Examples 1 and 2 copied substantially from source text; includes some verbatim copying	<i>Precalculus</i> , Sullivan, p. 398-399
p. 24	Examples 3 and 4 copied substantially from source text; includes some verbatim copying	<i>Precalculus</i> , Sullivan, p. 400-402
p. 25	Examples 5-7 copied substantially from source text; includes some verbatim copying	<i>Precalculus</i> , Sullivan, p. 402-404
p. 26	Problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 406-407
p. 29	Circle Trig homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 416
p. 34	Graphs & Transformations problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 429-430
p. 35	Tables are copied from Table 9 and Figure 93 of source text	<i>Precalculus</i> , Sullivan, p. 433
p. 37	Graphing the Tangent problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 437-438
p. 44	Inverse Sine problems are copied directly from source text	<i>Precalculus</i> , Sullivan, p. 469-470
p. 47	Values of Inverse Trig Functions Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 476-477
p. 51	Trigonometric Identities Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 483
p. 57-58	Sum and Difference Formulas Homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 492-493
p. 61-62	Double-angle problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 501-502
p. 65-66	Solving Trig Equations problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 510-511 and 518-519
p. 67	Applications Involving Right Triangle problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 531
p. 71-72	Law of Sines homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 539-540
p. 73-74	Law of Cosines homework problems copied directly from source text	<i>Precalculus</i> , Sullivan, p. 547

MAT 182 Trigonometry Spring 2010 (50 pages obtained from a class student)

This second set of materials was obtained from a student in Dr. Martinez's Spring 2010 MAT 182 class. These materials do not contain homework problems but appear to be handouts with examples to be used for taking class notes. A review of these materials show that the incorporated examples were also copied from the Sullivan *Precalculus* textbook. As noted in the Copyright Law Background section, basic formulas and/or symbols do not contain sufficient creativity to be copyrightable. Here, however, the copying is enough to create a significant risk that the compilation of the various formulae and examples used have been copied from the Sullivan textbook.

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COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OF TEXT	SOURCE INFORMATION
p. 1	Trig Functions of General Angles taken from the source text	<i>Precalculus</i> , Sullivan, p. 398 (Examples 1 and 2)
p. 5	Domain & Range Trig Functions	<i>Precalculus</i> , Sullivan, p. 413-414 (Example 2).
p. 6	Using even and odd properties	<i>Precalculus</i> , Sullivan, p. 414-415 (Example 3)
p. 9	Examples for graphing sine and cosine identical to source text.	<i>Precalculus</i> , Sullivan, p. 419 (Example 1) and 421 (Example 3)
p. 10	$y=2\cos x$ text and graph tracks source text	<i>Precalculus</i> , Sullivan, p. 422-423
p. 11	Examples for amplitude identical to source text	<i>Precalculus</i> , Sullivan, p. 425 (Example 6) and p. 426 (Example 7)
p. 12	Substantially copied from Table 9 and Figure 93 in source text	<i>Precalculus</i> , Sullivan, p. 433
p. 13	Bottom graphs ($y = \cot x$) appears to be a direct copy from the text	<i>Precalculus</i> , Sullivan, p. 436
p. 14	Top graph ($y = \csc x$) appears to be a direct copy from the text	<i>Precalculus</i> , Sullivan, p. 436
p. 15-17	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 461-469 (Examples 1-9)
p. 21-22	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 472-475 (Examples 1, 2, 4, 6)
p. 25-26	Copies Trig Identities examples from textbook and merely changes the order of appearance	<i>Precalculus</i> , Sullivan, p. 479 (Example 1)
p. 27-28	Copies "More examples" directly from textbook; summarizes "Guidelines for Establishing Identities" from box in textbook	<i>Precalculus</i> , Sullivan, p. 479-482 (Examples 2-8)
p. 31-32	Copies examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 488-490 (Examples 4-8)
p. 39-40	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 507-508 (Examples 1-4) and 514 (Example 1).
p. 41	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 414-515 (Examples 2-4)
p. 42	Copies problem examples directly from textbook	<i>Precalculus</i> , Sullivan, p. 528 (Examples 1 and 2)
p. 43	Word problem is an identical copy to Example 3 in textbook	<i>Precalculus</i> , Sullivan, p. 529 (Example 3)
p. 44	Law of Sines text and images are identical to textbook	<i>Precalculus</i> , Sullivan, p. 533
p. 45	"Solve the oblique triangle" text is virtually verbatim	<i>Precalculus</i> , Sullivan, p. 533
p. 46-48	Law of Sines problem examples copied directly from textbook	<i>Precalculus</i> , Sullivan, p. 534-537 (Examples 1-5)
p. 49-50	Law of Cosines problem examples are copied directly from textbook	<i>Precalculus</i> , Sullivan, p. 545 (Examples 1 and 2)

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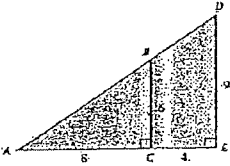
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Basic Arithmetic (MAT082)

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
32-33	Discrepancy: For the median and mode examples, the term "Juanita's Card Shop" is used. But, for the mean example, "Sarah's Card Shop" is used. In light of the evidence of other substantial copying, this is suggestive of copying a series of problems related to "Sarah's Card Shop" and changing "Sarah" to "Juanita" in some of the problems.	N/A
52, Nos. 27 and 28	<p>Problem No. 27, corresponds with Chapter 4.1, Problem No. 36</p> <p>Identical image:</p>  <p>Identical text:</p> <ul style="list-style-type: none"> • "the diagram below, AC represents the length of the line segment that starts at A and ends at C" • "we see that $AC = 8$" • "a. Find the ratio of BC to AC" • "b. What is the length AE?" • "c. Find the ratio of DE to AE." <p>Problem No. 28, corresponds with Chapter 4.1, Problem No. 33</p> <p>These questions are nearly identical, the Course Materials replace "rent" with "house payment", insert the phrase "living in a small house (not in Phoenix)" and change the dollar amount of the house payment/rent. The Course Materials use a different graph. The problems are approximately 80% identical, the problem in the Course Materials appears immediately after the problem above and both problems appear in the same section of the outside source. Thus, we conclude there is a strong likelihood this problem was copied.</p>	<p>Basic Mathematics Charles P. McKeague Page Numbers Not Available</p>
53, Nos. 29-32	<p>Problem No. 29, corresponds with Chapter 4.2, Problem No. 1</p> <p>Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text.</p> <p>"A car travels 220 miles in 4 hours. What's the rate of the car in miles per hour?"</p> <p>Problem No. 30, corresponds with Chapter 4.2, Problem No. 2</p> <p>Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text.</p> <p>"A train travels 360 miles in 5 hours. What's the rate of the train in miles per hour?"</p> <p>Problem No. 31, corresponds with Chapter 4.2, Problem No. 5</p> <p>Identical text except Course Materials add "Draw a picture demonstrating this" after the identical text.</p> <p>"The flow of water from a water faucet can fill a 3-gallon container in 15 seconds. Give the ratio of gallons to seconds as a rate in gallons per second."</p> <p>Problem No. 32, corresponds with Chapter 4.2, Problem No. 9</p> <p>Identical text</p> <p>"A car travels 95 miles on 5 gallons of gas. Give the ratio of miles to gallons as a rate in miles per gallon."</p>	<p>Basic Mathematics Charles P. McKeague Page Numbers Not Available</p>

2274286.1

ATTORNEY-CLIENT PRIVILEGED COMMUNICATION

GARRISON 0330

MCCCD/Martinez 01341

LEWIS
AND
ROCA
LLP
LAWYERS

Memorandum

October 28, 2010

Page 7

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
63, Nos. 5 and 6	<p><u>Problem No. 5, corresponds with Chapter 5, Problem No. 66</u> Identical text except for omission of "by a stake" in Course Materials "A wire from the top of a 24-foot pole is fastened to the ground by a stake 10 feet from the bottom of the pole. How long is the wire?"</p> <p><u>Problem No. 6, corresponds with Chapter 5, Problem No. 65</u> Identical text "A ladder is leaning against the top of a 15-foot wall. If the bottom of the ladder is 20 feet from the wall, how long is the ladder?"</p>	<p><i>Prealgebra</i> Alan R. Tussy R. David Gustafson p. 420</p>
77, Nos. 1, 11 and 14-16	<p><u>Problem No. 1, corresponds with Chapter 9, Figure 9-8</u> Identical text "How many cubic feet of water are needed to fill a spherical water tank with a radius of 15 feet?"</p> <p><u>Problem No. 11, corresponds with Chapter 9, Problem No. 50</u> Identical text "A classroom is 40 feet long, 30 feet wide, and 9 feet high. Find the number of cubic feet of air in the room."</p> <p><u>Problem No. 14, corresponds with Chapter 9, Problem No. 52</u> Identical text except Course Materials replace "advertised in a J.C. Penny catalog" with "in New Mexico" "The largest refrigerator advertised in a J.C. Penny catalog has a capacity of 25.2 cubic feet. How many cubic inches is this?"</p> <p><u>Problem No. 15, corresponds with Chapter 9, Problem No. 55</u> Identical text "The lifting power of a spherical balloon depends on its volume. How many cubic feet of gas will a balloon hold if it is 40 feet in diameter?"</p> <p><u>Problem No. 16, corresponds with Chapter 9, Problem No. 56</u> Identical text "A box of cereal measures 3 by 8 by 10 inches. The manufacturer plans to market a smaller box that measures 2.5 by 7 by 8 inches. By how much will the volume be reduced?"</p>	<p><i>Prealgebra</i> Alan R. Tussy R. David Gustafson p. 615 (No. 1) p. 620 (Nos. 11, 14-16)</p>

Precalculus Algebra Section Lecture Notes Fall 2010 (MAT 187)

This document was provided to us in Adobe PDF format that appears to have been created from a Word 2007 document. Due to the lack of any word problems in this work, and our limited access to potential source works, it was not possible for us to determine whether this particular work is a work of independent creation by Dr. Martinez or if she copied problems and equations from other works as is clearly the case in the other materials. We did, however, come upon a couple of irregularities within this document that are suspicious:

COURSE MATERIALS PAGE/QUESTION #	DISCREPANCY OR TEXT	SOURCE INFORMATION
p. 21	In Homework for Symmetry, the introductory text states "In problems 29-37..." but the problems immediately following the intro paragraph are numbered 1-5, which suggests that these problems were copied from another source and simply renumbered.	N/A
p. 22	Graphs in problems 6-28 are not presented in a consistent appearance	N/A

2274286.1

ATTORNEY-CLIENT PRIVILEGED COMMUNICATION

GARRISON 0331

MCCCD/Martinez 01342



Memorandum

October 28, 2010

Page 8

We cannot state definitively whether this particular work was created independently, nor is there sufficient evidence in the work itself for us to form an opinion that it was copied from other sources without permission. However, in light of the discrepancy found on page 21 and the history of copying evident in the other Course Materials, there is reason for concern. It may be helpful if the original Word document in which this work was created can be obtained from Dr. Martinez's workplace computer and reviewed.

ANALYSIS

To illustrate copyright law as applied to the Course Materials, we use a problem found in the Course Materials and in Basic Mathematics:

"The lifting power of a spherical balloon depends on its volume. How many cubic feet of gas will a balloon hold if it is 40 feet in diameter?"

The underlying equation for the word problem is the volume of a sphere -- $\frac{4}{3} \pi r^3$ -- a fact that cannot be described in any other way and thus not entitled to copyright protection. But, the word problem uses particular facts and word choice to test knowledge of the established mathematic formula. Therefore, although the equation is not copyrightable, the word problem is "original" and entitled to copyright protection.

Note that while the equation $\frac{4}{3} \pi r^3$ is not entitled to copyright protection by itself, if it were included in a compilation of other formulas, the compilation could be entitled to copyright protection. Accordingly, problem sets in the Course Materials that consist solely of equations could still be an infringement risk if they were copied as a "compilation" of equations from another source.

The word problems listed in the chart for the Basic Arithmetic Course Materials are substantially similar to word problems or examples found in other sources. Each of these word problems or examples likely possess the necessary originality for copyright protection. Accordingly, our review of the Course Materials indicates that they present a significant risk of infringement.

CONCLUSION

In order to avoid the risk of copyright infringement, Phoenix College should not allow Dr. Martinez's Course Materials to be distributed or used, and Phoenix College should immediately remove the Basic Arithmetic Course Materials from its website.

SDG/NE

ATTORNEY-CLIENT PRIVILEGED COMMUNICATION

2274286.1

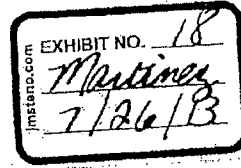
GARRISON 0332

MCCCD/Martinez 01343

EXHIBIT 20



VIA HANDDELIVERY

DR. ANNA SOLLEY
OFFICE OF THE PRESIDENT

December 9, 2010

Dr. Cleopatria Martinez
Mathematics Faculty Member
Phoenix College

Dear Dr. Martinez,

During the spring semester of 2010, I initiated an administrative evaluation based on Mr. Joe Sueyoshi's complaint, pursuant to section 3.7 of the Residential Faculty Policies Manual. By agreement, the Evaluation Team did not undertake its task until this fall semester.

I have reviewed the report of the Evaluation Team that was sent to me on November 17, 2010. The Evaluation Team which was advised by expert counsel, found that your work violated copyright. In addition, the Evaluation Team found that you were insubordinate and failed to follow District Legal Counsel's and my instructions and continued to request and persist in printing or copying unauthorized materials. The Evaluation Team agreed "that the administration should take appropriate disciplinary action following district policies in regards to this matter."

I have also considered our experiences under the current temporary arrangement in which Mr. Sueyoshi reviews materials you submit and may approve copying if they appear to be free of copyright and attribution problems. Given the Evaluation Team's findings and my lack of confidence in your willingness to follow the law as well as Maricopa policies and procedures due to your repeated copyright violations, I no longer consider the arrangement to be sufficient to protect the institution's legal and financial interests.

Pursuant to section 3.7.4 of the RFP, I now direct you to use only course materials approved by the department, that are available in the bookstore for sale to the students and that are authored by persons other than yourself. With department chair and Vice President for Academic Affairs approval, you may customize these materials for your students using traditional course packs or new online programs such as McGraw-Hill's "Create" (<http://create.mcgraw-hill.com/createonline/index.html#>) or McMillan's "Dynamic Books" (<http://dynamicbooks.vitalbook.com/>). All materials you use must be clearly attributed to authors other than yourself, and you must produce evidence of purchase or the author's permission to use them (e.g., a copy of the letter giving you a courtesy copy of the instructor's edition). If you wish to make copies of or from these materials, you must first produce written permission from the author and/or publisher to Mr. Sueyoshi. In addition, requests for the copying of these materials or your syllabi and quizzes/tests must be submitted in a timely manner (minimum of two work days notice) to Mr. Sueyoshi for approval.

MAIN CAMPUS
1202 West Thomas Road
Phoenix, AZ 85013
(602) 285-7433

PC DOWNTOWN CAMPUS
640 North 1st Avenue
Phoenix, AZ 85003
(602) 223-4000

GO FAR, CLOSE TO HOME.
phoenixcollege.edu

A Maricopa Community College.

MCCCD/Martinez01055

MCCCD/Martinez 01440

Dr. Cleopatria Martinez
December 9, 2010
Page 2

This direction is intended to communicate job duties to you within the meaning of Governing Board employment standard A4.3. In accordance with that section of the All Employee Policy Manual, willful and intentional violation of these instructions will be considered grounds for disciplinary action, up to and including termination of your employment.

Sincerely,



Anna Solley, Ed.D.
President

cc: Mr. Lee Combs
Ms. Casandra Kakar
Mr. Joe Sueyoshi

MCCCD/Martinez01056

MCCCD/Martinez 01441

EXHIBIT 21



NOTICE

Employee Name: Cleopatria Martinez Location: Phoenix College
Date Presented: October 18, 2012 Position: Residential Faculty Member

I. Corrective Action

Corrective Action Level:

- ☐ Initial Corrective Action Discussion
- ☒ Second Corrective Action Discussion
- ☐ ____ Hour Suspension without Pay, to be served from ____ 20__ through ____ 20__.
- ☐ Other: _____

Offense: This corrective action is being issued to you for unacceptable conduct related to the following:

Violation of MCCCCD Administrative Regulation 6.7.1 – Employment Standards: Willful and intentional violation of any state or federal law, applicable ordinance, MCCCCD Governing Board policy, or MCCCCD administrative regulation that affects the employee's ability to perform his or her job.

Violation of Administrative Regulation 1.12 – Cash Handling – Which covers all employees – Board-approved or part-time, cashiers offices, performing arts centers, athletic facilities, program offices, etc. that accept payment for any MCCCCD services or donations (including, but not limited to: tuition, fees, dues, event tickets, etc.). Cash is defined as coins, currency, checks, money orders, credit cards, electronic funds transfers, and all cash equivalents (including, but not limited to: tokens, gift cards, tuition waivers, parking tickets, stamps).

Violation of board employment standard A4.3., "Willful and intentional failure to perform job duties that have first been communicated to an employee and are within the employee's scope of employment." You willfully and intentionally failed to perform the duty I communicated to you in the specific written directive dated December 9, 2010 to "use only course materials approved by the department, that are available in the bookstore for sale to students and that are authorized by persons other than yourself. . . ."

Incident Description/Findings of Fact: (include the following information – time, place, date(s), of occurrence(s), persons present and impact on the District).

On December 9, 2010 I provided you with a specific directive related to your duties and responsibilities for use of course materials. That directive (copy attached) was in response to the findings of an Administrative Evaluation of you conducted during the fall semester of 2010 and delivered to me on November 17, 2010. In this evaluation the evaluation team found that your work violated copyright and that you were insubordinate and failed to follow District policy, Federal law,

Legal Counsel's advice, and my previous instructions and continued to print or copy unauthorized materials for use in your classes.

It has now come to my attention that on August 21 and August 23, 2012 you told students in your MAT 091 and MAT 151 class not to buy the approved text that you listed on the syllabus, but instead, you made copies of a colleague's materials at Staples and required your students to reimburse you for the cost (\$11 each). You provided no receipts. This transaction is a violation of my directive to you on December 9, 2010 and MCCC's cash handling rules as detailed in Administrative Regulation 1.12.

Prior Notifications:

<u>Level of Corrective Action</u>	<u>Date</u>	<u>Offense</u>
Initial Corrective Action	<u>12/9/10</u>	<u>Insubordination and failure to follow instructions regarding printing or copying unauthorized materials.</u>
Second Corrective Action	<u>N/A</u>	_____
Written Reprimand	<u>N/A</u>	_____
____ Hour Suspension without Pay	<u>N/A</u>	_____

Due Process:

Administrative Evaluation held 11/17/10

II. Performance Improvement Plan

1. Measurable/Tangible Goals: *I hereby remind you of my direction to you regarding your duties and responsibilities concerning the use of course materials, which remains in force and effect. Under this directive, you may only use course materials approved by the department, that are available in the bookstore for sale to students and that are authorized by persons other than yourself. With department chair and Vice President for Academic Affairs approval, you may customize these materials for your students using traditional course packs or new online programs such as McGraw-Hill's "Create or McMillan's "Dynamic Books." All materials you use must be clearly attributed to authors other than yourself, and you must produce evidence of purchase or author's permission to use them (e.g., a copy of a letter giving you a courtesy copy of the instructor's edition). If you wish to make copies of or from these materials, you must first produce written permission from the author and/or publisher to Mr. Sueyoshi. In addition, requests for copying of these materials or your syllabi and quizzes/tests must be submitted in a timely manner (minimum of two work days' notice) to Mr. Sueyoshi for approval.*
2. Training or Special Direction Provided: *A full copy of the December 9, 2010 directive is attached for your review and record.*
3. Interim Performance Evaluation: *No*
4. EAP: *Our Employee Assistance Program (EAP) Provider – MHN, can be reached confidentially to assist you toll-free, 24-hours per day at (800) 603-2970. This is strictly voluntary. Information regarding the EAP's services is available in Human Resources.*

5. Employee Input: *In addition, I recognize that you may have certain ideas to improve your performance. Therefore, I encourage you to provide your own Personal Performance Improvement Plan Input and Suggestions.*

III. Outcomes and Consequences

Compliance: *I will remain available to help you and discuss areas where you require additional support. If you refrain from further violations of this law and these District policies, no further corrective action will be taken.*

Non-Compliance: *This is your second notice regarding your failure to follow the specific instruction I provided to you to prevent further violations of district policy and federal law concerning copyright by printing or copying unauthorized materials. This is your first notice regarding your failure to follow MCCCCD policy regarding cash handling. Because you imposed charges on the students without authority to do so, you have a responsibility to reimburse the students from your own funds. You are hereby directed to do so by personal check, beginning immediately and continuing until all students who paid you are reimbursed. Students will be informed and will be expecting this payment. If you again violate the specific terms of my directive to you on December 9, 2010, as incorporated herein by reference and attachment, you will be subject to progressive corrective action up to and including termination of employment.*

Scheduled Review Date(s): *None*

IV. Employee Comments and/or Rebuttal

(Attach additional sheets if necessary)

Employee's Signature

Date

V. Employee Acknowledgment

I understand that this corrective action is intended to help me improve my performance. I understand that further violations of the District's standards of performance and conduct will lead to progressive corrective action up to and including termination of employment.

By my signature below I acknowledge that I have received a copy of this notification, that it has been discussed with me, and I understand its contents.

Employee's Signature

Date

Anna Solley
President
Phoenix College

Date

EXHIBIT 22

4/4/13

Maricopa Community College District Mail - Copy of Message sent to MAT151 class



From: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Copy of Message sent to MAT151 class

1/1

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>
Reply-To: joe.sueyoshi@phoenixcollege.edu
To: Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Thu, Apr 4, 2013 at 3:15 PM

Here it is:

On Nov 28, 2012, at 12:06 PM, "Joe Sueyoshi" <joe.sueyoshi@phoenixcollege.edu> wrote:

Dear Student,

It has come to my attention that Dr. Martinez, Math Faculty, may have sold you instructional materials. She is not authorized to have such materials printed, and District cash handling regulations prohibit her from selling them to you. If you made this purchase, she is required to refund the purchase price to you via check before the end of the Fall semester, 2012. Please contact Dr. Martinez directly to receive your refund. If you know someone in the course who has withdrawn or who may not have received this e-mail, please let him or her know of the refund and have them contact Dr. Martinez. If you have not received the refund via check by semester's end, please reply to this e-mail.

Her contact information: cleopatria.martinez@phoenixcollege.edu 602/285-7390; Office: Room A-243

Thanks,

joe

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

5/13/13

Maricopa Community College District Mail - Re: Phoenix College: MAT091 19577 message



Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Re: Phoenix College: MAT091 19577 message

4 messages

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Tue, Dec 11, 2012 at 12:52 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Joseph Hobson <jos2136060@maricopa.edu>

Cc: Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Typically, the grievance process starts with a conversation with the instructor to discuss the issue(s). However, if this was not possible, then the grievance process goes to me, the department chair. Since you have filed paperwork directly to the school (which I assume is our administration), then either the outcome will come from administration or the grievance will be return to me to handle it. I have not received any notice of yet. So, I must assume at the moment that the resolution will come from administration.

I would suggest asking the person(s) that you filed the grievance paperwork with for a timeline for resolution.

joe

On Tue, Dec 11, 2012 at 11:16 AM, Joseph Hobson <jos2136060@maricopa.edu> wrote:

Dear Mr. Sueyoshi,

I just completed Dr. Martinez's 091 class and I am sure I have a failing grade. I have no aptitude towards mathematics and have struggled with it in the past and I feel like this class was counterproductive to say the least. I have filed grievance paperwork with the school and am curious what the outcome is going to be. The outcome of this class will determine my financial eligibility next semester and I am looking for a resolution to this problem.

I would like the opportunity to retake this class with a different instructor if possible. Also if there is anyway around the impending failing grade that is going to affect my academic future, please let me know.

Looking forward to hearing from you,

Joe Hobson

On Wed, Nov 28, 2012 at 12:05 PM, Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu> wrote:

Dear Student,

It has come to my attention that Dr. Martinez, Math Faculty, may have sold you instructional materials. She is not authorized to have such materials printed, and District cash handling regulations prohibit her from selling them to you. If you made this purchase, she is required to refund the purchase price to you via check before the end of the Fall semester, 2012. Please contact Dr. Martinez directly to receive your refund. If you know someone in the course who has withdrawn or who may not have received this e-mail, please let him or her know of the refund and have them contact Dr. Martinez. If you have not received the refund via check by semester's end, please reply to this e-mail.

Her contact information: cleopatria.martinez@phoenixcollege.edu 602/285-7390; Office: Room A-243

Thanks,

joe

5/13/13

Maricopa Community College District Mail - Re: Phoenix College: MAT091 19577 message

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Wed, Dec 12, 2012 at 3:34 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Lee Combs <lee.combs@domail.maricopa.edu>

Cc: Kelly Loucks <kelly.loucks@phoenixcollege.edu>

FYI: Another student who has been refused the refund. Is there a Plan B? May I suggest a suspension? RFP:
3.13. Suspension of a Faculty Member

joe

----- Forwarded message -----

From: **Marquisporter_2010** <Marquisporter_2010@yahoo.com>

Date: Tue, Dec 11, 2012 at 9:24 PM

Subject: RE: Phoenix College: MAT091 19577 message

To: joe.sueyoshi@phoenixcollege.edu

Excuse me sir...not only did she refuse to give me the refund. She said she wasn't because you guys were just messing with her. But also I requested to be withdrew from her class repeatedly more than a month ago and she hasn't last time I checked which was Thursday. I couldn't learn in her class as she constantly was missing from the class (a few test as well) she also verbally insulted me as if I were a child. I just thought I'd share that with you sir. Have a good night.

From my Android phone on T-Mobile. The first nationwide 4G network.

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu> wrote:

Dear Student,

It has come to my attention that Dr. Martinez, Math Faculty, may have sold you instructional materials. She is not authorized to have such materials printed, and District cash handling regulations prohibit her from selling them to you. If you made this purchase, she is required to refund the purchase price to you via check before the end of the Fall semester, 2012. Please contact Dr. Martinez directly to receive your refund. If you know someone in the course who has withdrawn or who may not have received this e-mail, please let him or her know of the refund and have them contact Dr. Martinez. If you have not received the refund via check by semester's end, please reply to this e-mail.

Her contact information: cleopatria.martinez@phoenixcollege.edu 602/285-7390; Office: Room A-243

Thanks,

joe

[Quoted text hidden]

5/13/13

Maricopa Community College District Mail - Re: Phoenix College: MAT091 19577 message

Lee Combs <lee.combs@domail.maricopa.edu>

Wed, Dec 12, 2012 at 3:44 PM

Reply-To: lee.combs@domail.maricopa.edu

To: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Cc: Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Jim, what do you think?

[Quoted text hidden]



Lee Combs: General Counsel

2411 West 14th Street, Tempe AZ 85281

phone | 480-731-8878 • fax | 480-731-8890

e-mail | lee.combs@domail.maricopa.edu

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Thu, Apr 4, 2013 at 3:20 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Cc: Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Contained in the string of messages is the the message I sent to the MAT091 class.

It also contains the name of one of several students who filed a complaint and worked with Wilbert. Wilbert may also have some students that can be contacted by Judy (if the one I have in mind is not available).

joe

——— Forwarded message ———

From: **Joseph Hobson** <jos2136060@maricopa.edu>

Date: Tue, Dec 11, 2012 at 11:16 AM

Subject: Re: Phoenix College: MAT091 19577 message

To: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Dear Mr. Sueyoshi,

I just completed Dr. Martinez's 091 class and I am sure I have a failing grade. I have no aptitude towards mathematics and have struggled with it in the past and I feel like this class was counterproductive to say the least. I have filed grievance paperwork with the school and am curious what the outcome is going to be. The outcome of this class will determine my financial eligibility next semester and I am looking for a resolution to this problem.

I would like the opportunity to retake this class with a different instructor if possible. Also if there is anyway around the impending failing grade that is going to affect my academic future, please let me know.

Looking forward to hearing from you,

Joe Hobson

On Wed, Nov 28, 2012 at 12:05 PM, Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu> wrote:

Dear Student,

5/13/13

Maricopa Community College District Mail - Re: Phoenix College: MAT091 19577 message

It has come to my attention that Dr. Martinez, Math Faculty, may have sold you instructional materials. She is not authorized to have such materials printed, and District cash handling regulations prohibit her from selling them to you. If you made this purchase, she is required to refund the purchase price to you via check before the end of the Fall semester, 2012. Please contact Dr. Martinez directly to receive your refund. If you know someone in the course who has withdrawn or who may not have received this e-mail, please let him or her know of the refund and have them contact Dr. Martinez. If you have not received the refund via check by semester's end, please reply to this e-mail.

Her contact information: cleopatria.martinez@phoenixcollege.edu 602/285-7390; Office: Room A-243

Thanks,

joe

[Quoted text hidden]

5/13/13

Maricopa Community College District Mail - \$11 Refund



Kelly Loucks <kelly.loucks@phoenixcollege.edu>

\$11 Refund

1 message

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Tue, Dec 11, 2012 at 1:50 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Cleopatria Martinez <cleopatria.martinez@phoenixcollege.edu>

Cc: Lee Combs <lee.combs@domail.maricopa.edu>, Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Sharon Halford <sharon.halford@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>

I have received a complaint from one of your former students concerning the \$11 refund. As per the corrective action and with consultation with District Legal, the student is to receive the \$11 reimbursement. The student can provide a receipt, but returning the booklet is not a condition of receiving the refund. This type of conflict is one of the reasons why there are District cash handling policies.

Failure to comply with students' requested refunds may be interpreted as insubordination. I suggest you proceed accordingly.

joe

—
Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

4/4/13

Maricopa Community College District Mail - Fwd: MAT091 & MAT151 Materials



Fwd: MAT091 & MAT151 Materials

Fwd: MAT091 & MAT151 Materials

4/4/13 Joe

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>
 Reply-To: joe.sueyoshi@phoenixcollege.edu
 To: Casandra Kakar <casandra.kakar@phoenixcollege.edu>
 Cc: Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Thu, Apr 4, 2013 at 3:22 PM

Here is one message I sent on Tim's behalf (he was "Bcc'd").

joe

----- Forwarded message -----

From: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>
 Date: Thu, Dec 13, 2012 at 4:12 PM
 Subject: MAT091 & MAT151 Materials
 To: Cleopatria Martinez <cleopatria.martinez@phoenixcollege.edu>
 Cc: Lee Combs <lee.combs@domail.maricopa.edu>, Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>, Sharon Halford <sharon.halford@phoenixcollege.edu>

This is a notice before leaving for Winter Break to remind you that the author of the MAT091 & MAT151 materials that you used this semester will not grant you permission to use the materials in future semesters.

As per the Directive, which supercedes any department policies on textbook usage, you will not be able to use any copies of these materials in future semesters as a result.

As for the reasons the author will not grant you permission, I was told it is because you lied to the author when you stated how the materials were to be distributed to the students. In fact, when asked directly over the phone from the author, you denied that you were selling the materials directly to students for \$11. Since this is not the first time you have lied, I find the author's reasons as credible. However, if this is a distortion of the facts, I'm sure you can ask the author again for permission and get a response.

If the reason you are not permitting the \$11 refund to students this semester is the fact that you plan to use these materials in future semesters, that will not be an issue anymore. I have had a couple of students now requesting refunds. I've been told that at least one student has resorted to calling you with inappropriate adjectives, but try not to take it personally. However, this is one of the consequences of denying students their proper refund. If your concerns are for students, please respect their wishes and give them their refunds.

Thanks,

joe

Joe Sueyoshi
 Phoenix College
 Math Department Chair
 602/285-7885

4/4/13

Maricopa Community College District Mail - Fwd: MAT091 & MAT151 Materials

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

Casandra Kakar <casandra.kakar@phoenixcollege.edu>
Reply-To: casandra.kakar@phoenixcollege.edu
Draft To: sbalch@laborlawyers.com

Thu, Apr 4, 2013 at 3:27 PM

----- Forwarded message -----

From: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>
[Quoted text hidden]

5/13/13

Maricopa Community College District Mail - Response So Far



Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Response So Far

1 message

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Mon, Jan 7, 2013 at 6:14 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Lee Combs <lee.combs@domail.maricopa.edu>, Jim Bowers <james.bowers@domail.maricopa.edu>, Casandra Kakar <casandra.kakar@phoenixcollege.edu>, Judy Castellanos <judy.castellanos@domail.maricopa.edu>

Cc: Kelly Loucks <kelly.loucks@phoenixcollege.edu>, Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>, Anna Solley <anna.solley@phoenixcollege.edu>

Kelly is asking the following two questions to the students:

- (1) Did you purchase the materials directly from the instructor? If yes,
- (2) Did you receive a refund from the instructor?

Of the 17 students called so far in her MAT091 class, only 6 have responded (messages were left for those who did not respond). None of the 6 have received a refund. Kelly can correct me if I misspeak, but it appears in 5 of the cases so far, the student did not ask for a refund (even though they received the message) and also, Dr. Martinez did not offer the refund. In one case, it appears there was a discussion between a student and the instructor in her office, after which the refund discussion was dropped. Of course, the matter should not have been up for discussion, but I guessing that she plead her case and convinced the student.

I guess the question to ponder is whether it matters or not that the instructor may have pleaded her case to the students, individually or as a group, instead of just handing out checks. Also, manipulated or otherwise, does it matter whether or not the students wanted to receive the checks?

Once the entire class is notified, we can contact them to send e-mails to confirm. However, so far none contacted have received a check..

joe

--

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

5/13/13

Maricopa Community College District Mail - Regarding MAT151 Class #19036 From Fall 2012



Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Regarding MAT151 Class #19036 From Fall 2012

2 messages

Kelly Loucks <kelly.loucks@phoenixcollege.edu>
 Reply-To: kelly.loucks@phoenixcollege.edu
 To: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Tue, Jan 8, 2013 at 6:16 PM

Good afternoon,

I have called 24 students who were either withdrawn or received a grade in MAT151 #19036 during the Fall 2012 semester. I made 8 contacts and only 1 of the students received a refund from the instructor for the class materials. 7 Students did not receive refunds.

A couple of the students were hostile on the phone and hung up before I could complete the calls.

I received 1 follow up message from the MAT091 class yesterday from a student who did not receive a refund.

Let me know if you need any additional information and let me know if I need to contact dropped students.

Thank you,

Kelly

—
 Kelly Loucks
 Secretary
 Mathematics Department
 Phoenix College
 602-285-7151

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Tue, Jan 8, 2013 at 7:00 PM

Reply-To: joe.sueyoshi@phoenixcollege.edu
 To: Lee Combs <lee.combs@domail.maricopa.edu>, Jim Bowers <james.bowers@domail.maricopa.edu>, Judy Castellanos <judy.castellanos@domail.maricopa.edu>, Casandra Kakar <casandra.kakar@phoenixcollege.edu>
 Cc: Anna Solley <anna.solley@phoenixcollege.edu>, Kelly Loucks <kelly.loucks@phoenixcollege.edu>, Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>

Please read Kelly's report in this message.

Kelly tried to contact students in the 2nd class. Only 1 of 8 that Kelly talked to received the refund. If you wish, we can confirm whether or not the one student was the President's student-worker. If it is (and we don't know who the President's student-worker is), that would say it all.

I'm not sure if the students who were hostile on the phone was due to their experience in Dr. Martinez' class or the math department/college's inability to assist them with refunds.

In any case, Kelly & I believe there is no need to e-mail students to confirm whether or not refunds were

<https://mail.google.com/mail/u/0/?ui=2&ik=1c4ddc18dd&view=pt&q=martinez&psize=20&pmr=100&pdr=50&search=apps&th=13c1ce1b43c62bd9>

1/2

5/13/13

Maricopa Community College District Mail - Regarding MAT151 Class #19036 From Fall 2012

received. Kelly has a list of the names as evidence, if you wish to contact them by e-mail yourself to confirm. If you do, you would get a better sense of the indifference or hostility from the students and what a negative experience it has been for Kelly to do this.

We suspect that even if we e-mail them now (if they respond at all or the e-mail addresses are active), there will either be an indifference to respond (just move on) or hostility about having to respond. Of course, I don't think Dr. Martinez will be producing many copies of checks either. If she does, we can easily confirm their validity.

The students may respond better, if they know it's District Legal that is calling and they are notified that their checks are in the mail.

It's safe to say that, even with a small sample size, she did not give refunds to the majority of the students who purchased the materials. She may argue that the students did not want a refund, but that was NOT a condition for her to provide refunds, nor was the return of the materials a condition as she stated to at least one student.

In any case, you can send the e-mail to Dr. Martinez to give her an "opportunity" to respond about the refund checks. She will likely delay or refuse to respond.

Should I be planning classes for Dr. Martinez next week? Let me know ASAP.

Thanks,

joe.

[Quoted text hidden]

—

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885

5/13/13

Maricopa Community College District Mail - Concerning Cleopatria's Classes



Kelly Loucks <kelly.loucks@phoenixcollege.edu>

Concerning Cleopatria's Classes

2 messages

Kelly Loucks <kelly.loucks@phoenixcollege.edu>
Reply-To: kelly.loucks@phoenixcollege.edu
To: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Wed, Jan 9, 2013 at 12:45 PM

Good afternoon,

I have attached the scans of my notes from calling Cleopatria's classes to confirm if students had received refunds. My notes are written on the papers. Out of 52 total contacts between messages and speaking with students only 1 had confirmed receiving the refund. 20 others had indicated they had not received a refund from the instructor.

Please let me know if you have any questions.

Thank you,

Kelly

—
Kelly Loucks
Secretary
Mathematics Department
Phoenix College
602-285-7151

2 attachments

MAT091 class 19577.pdf
1214K

MAT151 class 19036.pdf
1084K

Kelly Loucks <kelly.loucks@phoenixcollege.edu>
Reply-To: kelly.loucks@phoenixcollege.edu
To: Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>
Cc: Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Thu, Jan 24, 2013 at 9:21 AM

Here is the student contact info I scanned.


Kelly
[Quoted text hidden]

2 attachments

MAT091 class 19577.pdf
1214K

5/13/13

Maricopa Community College District Mail - Concerning Cleopatria's Classes

 **MAT151 class 19036.pdf**
1084K

Class Roster

Page 1 of 1

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Class Roster

Course ID: 117906 College Algebra/Functions (Lecture)
 Catalog: MAT 151
 Class Nbr: 19036 Section 0010
 Instructors: Martinez, Cleopatra

Phoenix College
 Fall 2012
 Dynamic Dated

Class Roster Details			
Total Students:	50	*Enrollment Status:	All
		Detail	

		Name	Grade	Units	Program
1	35222299	Abrego, Leslie Damara	Graded	4.00	Degree and Cert Programs
2	35499968	Alontably, Fatima A (Dropped)	Graded	0.00	Degree and Cert Programs
3	35513283	Alvarado, Alexxas Alexandrya	Graded	4.00	Degree and Cert Programs
4	35506030	Aranda Acosta, Humberto	Graded	4.00	Degree and Cert Programs
5	35452099	Banks, Ashley Alicia (Dropped)	Graded	0.00	Degree and Cert Programs
6	35518817	Barde, Jordan (Dropped)	Graded	0.00	Degree and Cert Programs
7	35435299	Bishop, Kait Marie	Graded	4.00	Degree and Cert Programs
8	35672967	Breidenbach, Jacob Aiden	Graded	4.00	Degree and Cert Programs
9	35300973	Caraveo, Veronica M	Graded	4.00	Degree and Cert Programs
10	35546705	Do, Shon Le	Graded	4.00	Degree and Cert Programs
11	35432478	Dominguez, Aviana Jasmine	Graded	4.00	Degree and Cert Programs
12	35286877	Duran Hernandez, Xochitl (Dropped)	Graded	0.00	Degree and Cert Programs
13	35708119	Fernandez, Jessica Haley	Graded	4.00	Degree and Cert Programs
14	35387838	Floros, May Ann B (Dropped)	Graded	0.00	Degree and Cert Programs
15	35495546	Garcia-Torres, Erik Noe	Graded	4.00	Degree and Cert Programs
16	35245494	Gibson, Lashar T (Dropped)	Graded	0.00	Degree and Cert Programs
17	35470875	Gooding, Dominique (Dropped)	Graded	0.00	Degree and Cert Programs
18	35418917	Grijalva, Maritza Avalos	Graded	4.00	Degree and Cert Programs
19	35646263	Hardge, Alysha Chantelle (Dropped)	Graded	0.00	Degree and Cert Programs
20	35281061	Hasson, Hanan (Dropped)	Graded	0.00	Degree and Cert Programs
21	35330082	Hoyos, Andrea	Graded	4.00	Degree and Cert Programs
22	35390463	Jurnsin, Jesse Alan (Dropped)	Graded	0.00	Degree and Cert Programs
23	35337439	Knoll Brooks, Alan	Graded	4.00	Degree and Cert Programs
24	35675254	Lillard, Aleishia Teirshiana	Graded	4.00	Degree and Cert Programs
25	350812013	Lopez, Rachelle Lydia (Dropped)	Graded	0.00	Degree and Cert Programs
26	35276728	Lopez, Stephanie Diann (Dropped)	Graded	0.00	Degree and Cert Programs
27	35566351	Luster, Jeddiah	Graded	4.00	Degree and Cert Programs
28	35526105	Maxey, Tyler Jenise	Graded	4.00	Degree and Cert Programs
29	35533812	McAuley, Jason C	Graded	4.00	Degree and Cert Programs
30	35448575	Menke, Roby Denise	Graded	4.00	Degree and Cert Programs
31	35342056	Monroe, ShaRyse	Graded	4.00	Degree and Cert Programs
32	35569693	Mora, Yundia Anahi	Graded	4.00	Degree and Cert Programs
33	35432195	Muro, Yohara	Graded	4.00	Degree and Cert Programs
34	35493285	Navas Cardoza, Fredy	Graded	4.00	Degree and Cert Programs
35	35383800	Neal, Bryan Keith (Dropped)	Graded	0.00	Degree and Cert Programs
36	35459311	Nettles-Boyd, Kiara J	Graded	4.00	Degree and Cert Programs
37	35805318	Olivarez, Blanca (Dropped)	Graded	0.00	Degree and Cert Programs
38	35525695	Parra Hernandez, Laura Yarely	Graded	4.00	Degree and Cert Programs
39	33714304	Perez, Ismael Valdez	Graded	4.00	Degree and Cert Programs
40	33230744	Pokhias, Jared Keith (Withdrawn)			Degree and Cert Programs
41	35394421	Ponce, Jasus	Graded	4.00	Degree and Cert Programs
42	35760615	Quintana, Anahi	Graded	4.00	Degree and Cert Programs
43	35559058	Raskin, Kyndrah Yvonne	Graded	4.00	Degree and Cert Programs
44	35493317	Rodriguez, Berly (Dropped)	Graded	0.00	Degree and Cert Programs
45	35529325	Ruiz Soto, Salvador (Withdrawn)			Degree and Cert Programs
46	35521259	Saenz, Juan Carlos (Dropped)	Graded	0.00	Degree and Cert Programs
47	35483556	Serrano, Christine Monique	Graded	4.00	Degree and Cert Programs
48	35722393	Vasquez, Vincent Thomas (Dropped)	Graded	0.00	Degree and Cert Programs
49	33335226	Vazquez, Moises P (Withdrawn)			Degree and Cert Programs
50	35362761	Weaver, Isalah (Dropped)	Graded	0.00	Degree and Cert Programs

Return to Search | Notify

Contracts
 24/8 + 1
 Need refund
 Received refund

623-336-1933

✓ indicate a message was left or I spoke to the student

- indicates student paid \$11 to Cleopatra and did not receive refund
 + indicates student received refund

Not charged - withdrawn

Class Roster

Page 1 of 2

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Class Roster

Course ID: 117881 Introductory Algebra (Lecture)
 Catalog: MAT 091
 Class Nbr: 19577 Section 0031
 Instructors: Martinez,Cleopatia

Phoenix College
 Fall 2012
 Dynamic Dated

Class Roster Details			
Total Students:	65	*Enrollment Status:	All
		Detail	

Customize [Add] [View All] [Print] [Export] [Import] [Reset]			
ID	Name	Grade	Units
✓ 35273370	Aguayo,Dulce Maria	Graded	4.00
2 35586940	Aklur,Makluma (Dropped)	Graded	0.00
3 33230377	Almanza,Anthony Thomas (Dropped)	Graded	0.00
4 35658680	Alvarado Jr,John Edward (Dropped)	Graded	0.00
5 35727772	Basham,Randall Aaron (Dropped)	Graded	0.00
6 35307596	Bond,Marcus Lorenzo (Dropped)	Graded	0.00
✓ 35357312	Borbolla,Jesus	Graded	4.00
✓ 35432455	Carvajal,Corina Nora	Graded	4.00
✓ 35632669	Charmack,Garry (Withdrawn)		
10 30523635	Clay,Jason Charles (Dropped)	Graded	0.00
11 35509452	Corrales,Amanda (Dropped)	Graded	0.00
✓ 3568201	Dalbey,Mark Henry	Graded	4.00
✓ 35619228	Echols,Dominick C	Graded	4.00
14 35706037	Egelhof,Laura Paulann	Graded	4.00
15 35656982	Flores,Reymond Michael (Dropped)	Graded	0.00
16 35318699	Galvan,Raquel Lynn (Dropped)	Graded	0.00
✓ 32283846	Gamble,Sieven R (Withdrawn)		
✓ 35292031	Garnes,Melissa (Withdrawn)		
✓ 35725373	Garcia,Alejandra	Graded	4.00
✓ 30240664	Gonzalez,Joe Albert	Graded	4.00
21 30362925	Grijalva,Richard (Dropped)	Graded	0.00
22 35263230	Haile,Sara Ammanuel (Dropped)	Graded	0.00
23 30323781	Harvey,Jeremy W (Dropped)	Graded	0.00
24 35626596	Hazim,Tiba (Dropped)	Graded	0.00
✓ 35716109	Hernandez,Stephanie	Graded	4.00
26 35728313	Hidalgo Cruz,Juan Carlos (Dropped)	Graded	0.00
27 35535026	Higuera,Yazmin Adanelli (Dropped)	Graded	0.00
✓ 35241708	Hobson,Joseph P	Graded	4.00
✓ 31292333	Kipros,Michael A	Graded	4.00
✓ 35684222	Large,Alexis Nicole (Withdrawn)		
✓ 35356806	Lopez,Yesenia V	Graded	4.00
32 35692495	Maciel-Christensen,Anyssa M (Dropped)	Graded	0.00
33 35329119	Majeed,Nadia (Dropped)	Graded	0.00
✓ 35625295	Malaeey,Mukhammad (Withdrawn)		
✓ 35719690	Mancera Santos,Daisy (Dropped)	Graded	0.00
✓ 35291653	Martinez,Jessica Marie (Withdrawn)		
37 35638604	Matulesay,Margie (Dropped)	Graded	0.00
38 30301154	McBreen,Shirley A (Dropped)	Graded	0.00
39 35524437	Montenegro,Emily (Dropped)	Graded	0.00
40 35434852	Morales,Adrian Alberto (Dropped)	Graded	0.00
✓ 32808098	Morales,Suzanne T	Graded	4.00
✓ 35693489	Oniveros Ortiz,Maira (Dropped)	Graded	0.00
✓ 30374642	Palti,Elizabeth Anne (Dropped)	Graded	0.00
✓ 35586846	Popy,Rabeya A (Dropped)	Graded	0.00
✓ 35510686	Porter,Marquia Shaquille (Withdrawn)		
46 30738288	Powell,Nicole (Dropped)	Graded	0.00
47 35731017	Ramirez,Angelique Rose (Dropped)	Graded	0.00
48 35626007	Ramirez,Paulicia (Dropped)	Graded	0.00
✓ 30377864	Rarrick,Angela Cathenne	Graded	4.00
50 34581758	Robles,Gonnieve Irene (Dropped)	Graded	0.00
51 35743830	Robles,Gabriela (Dropped)	Graded	0.00
✓ 35734311	Rodriguez Rangel,Roman Eduardo (Withdrawn)		
✓ 35664315	Rosas Granados,Maria	Graded	4.00
54 35401923	Salas-Salazar,Aidel (Dropped)	Graded	0.00
✓ 35446006	Salazar,Grisel Celene	Graded	4.00
✓ 35704272	Sallinas,Angelina	Graded	4.00
✓ 35875661	Sanchez,Marisol Rocha	Graded	4.00
✓ 35579974	Sandoval,Gabriela	Graded	4.00
✓ 30341540	Scott,Antwane K (Withdrawn)		
✓ 34686430	Sheeran,Kimberly E (Withdrawn)		
61 35723138	Sims,Tyler Garmaine (Dropped)	Graded	0.00
✓ 35729025	Soldano,Theresa	Graded	4.00
✓ 35301410	Valdez,Juliane	Graded	4.00

Contacts 28/12
 Need refund

Offered, Not Accepted

✓ indicates a message was left or I spoke to the student

- indicates student paid \$11 to Cleopatia and did not receive refund

No phone

Wrong

Error

Wrong phone

5/13/13

Maricopa Community College District Mail - Fwd: refund of the book i purchased from you



Kelly Loucks <kellyloucks@phoenixcollege.edu>

Fwd: refund of the book i purchased from you

message

Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Thu, Jan 17, 2013 at 10:53 AM

Reply-To: joe.sueyoshi@phoenixcollege.edu

To: Lee Combs <lee.combs@domain.maricopa.edu>, Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Cc: Kelly Loucks <kelly.loucks@phoenixcollege.edu>, Wilbert Nelson <wilbert.nelson@phoenixcollege.edu>

Update on Elizabeth Patti: Dr. Martinez is still making the refund conditional on returning of the text (see message). I thought it was agreed that this was NOT the case.

joe

----- Forwarded message -----

From: **Elizabeth Patti** <eli2083605@maricopa.edu>

Date: Thu, Jan 17, 2013 at 10:00 AM

Subject: Fwd: refund of the book i purchased from you

To: joe.sueyoshi@phoenixcollege.edu

Good morning Joe, Last night I got this email from Dr. Martinez, and in it, she pretty much is saying that "in exchange for the math book, I will get the \$11. check"

Elizabeth A. Patti

----- Forwarded message -----

From: **Cleopatria Martinez** <cleopatria.martinez@phoenixcollege.edu>

Date: Wed, Jan 16, 2013 at 7:16 PM

Subject: Re: refund of the book i purchased from you

To: Elizabeth Patti <eli2083605@maricopa.edu>

Hi Elizabeth, 1-16-13

Please come to Phoenix College so that I can give you a check for \$11 in exchange for the book you no longer wish to have.

On monday and wednesday I'll be in my office 8:30-10am and in A101 from 10-12.

On tuesday and thursday I'll be in B210 from 7:30-10am, in A202 from 10-11am, and in my office from 11:15 until 12:00.

You are welcome to a free electronic copy of the MAT 091 material which I have included with this email.

However, it cost \$11 to have the copy printed and bound. Since you wish to have your money back, you should return the hard copy. I think you can understand that I never intended to purchase your textbook for you, but if you did not want a hard copy I still wanted you to have a copy and the electronic copy is attached.

Have a nice day,

Dr .C. Martinez

----- Forwarded message -----

From: **Elizabeth Patti** <eli2083605@maricopa.edu>

Date: Fri, Dec 7, 2012 at 6:31 PM

Subject: Re: refund of the book i purchased from you

5/13/13

Maricopa Community College District Mail - Fwd: refund of the book i purchased from you

To: cleopatria.martinez@phoenixcollege.edu

okay thank you

Elizabeth A. Patti

On Fri, Dec 7, 2012 at 10:22 AM, Cleopatria Martinez <cleopatria.martinez@phoenixcollege.edu> wrote:

Hi Elizabeth,

Please come by my office Monday sometime between 9am and noon. Have a lovely weekend.

Dr. Martinez

On Thu, Dec 6, 2012 at 6:14 PM, Elizabeth Patti <eli2083605@maricopa.edu> wrote:

Dear Dr. Martinez;

At the beginning of this semester (Fall 2012) I purchased a book from you for \$11.00, I was given a letter today stating that if any student purchased a book from you that we are intitled to a refund. I went by your office earlier today, but you must of just left, so I was instructed to email you, that way we can set up a day/time that works for you so that I can get the money for the book. I understand it will be in check form, if that is true could you make the check out to my husband Michael Patti, since he was the one that handed you the money.

Thank you,

Elizabeth Patti

P.S. I am enrolled in your Math Tutoring Class for next semester as well.
Have a Merry Christmas and a safe New Year.. =)

Elizabeth A. Patti

Joe Sueyoshi
Phoenix College
Math Department Chair
602/285-7885



MAT091 Fall 2012 .docx
1201K

4/4/13

Maricopa Community College District Mail - Fwd: Documentation of Refunds to Students


casandra.kakar@phoenixcollege.edu

Fwd: Documentation of Refunds to Students

Line 1400

Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Tue, Jan 22, 2013 at 9:52 AM

Reply-To: casandra.kakar@phoenixcollege.edu

To: Anna Solley <anna.solley@phoenixcollege.edu>, Lee Combs <lee.combs@domail.maricopa.edu>, Jim Bowers <james.bowers@domail.maricopa.edu>

Anna,

FYI, I have not received documentation from Dr. Martinez. The deadline was Friday, January 18, 2013.

Casandra

----- Forwarded message -----

From: Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Date: Fri, Jan 11, 2013 at 3:56 PM

Subject: Fwd: Documentation of Refunds to Students

To: Jim Bowers <james.bowers@domail.maricopa.edu>, Judy Castellanos <judy.castellanos@domail.maricopa.edu>, Anna Solley <anna.solley@phoenixcollege.edu>

----- Forwarded message -----

From: Casandra Kakar <casandra.kakar@phoenixcollege.edu>

Date: Fri, Jan 11, 2013 at 3:55 PM

Subject: Documentation of Refunds to Students

To: Cleopatria Martinez <cleopatria.martinez@phoenixcollege.edu>

Cc: Anna Solley <anna.solley@phoenixcollege.edu>, Lee Combs <lee.combs@domail.maricopa.edu>, Joe Sueyoshi <joe.sueyoshi@phoenixcollege.edu>

Dr. Martinez,

On October 18, 2012 Dr. Solley and I met with you regarding a Second Corrective Action Discussion.

In Section III. Outcomes and Consequences a specific directive was outlined:

Because you imposed charges on your students without authority to do so, you have the responsibility to reimburse the students from your own funds. You are hereby directed to do so by personal check, beginning immediately and continuing until all students who paid you are reimbursed.

Unfortunately, I am learning not all students have been reimbursed as of Wednesday, January 9, 2013. At this point, I am requesting copies of the cashed personal checks you have written to your students. This will require front and back copies of the processed checks you have written to each student. The deadline for the check copies is Friday, January 18, 2013.

Best regards,
Casandra